CAMBRIDGE INTERNATIONAL EXAMINATIONS

Cambridge Ordinary Level

MARK SCHEME for the May/June 2015 series

4037 ADDITIONAL MATHEMATICS

4037/12 Paper 1, maximum raw mark 80

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Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied
www	without wrong working

1	$k^2-4(2k+5)\quad(<0)$ $k^2-8k-20\quad(<0)$	M1	use of b^2-4ac, (not as part of quadratic formula unless isolated at a later stage) with correct values for a, b and c Do not need to see < at this point
	$(k-10)(k+2)\quad(<0)$	M1	attempt to obtain critical values
	critical values of 10 and –2	A1	correct critical values
	$-2<k<10$	A1	correct range
	Alternative 1:		
	$\frac{\mathrm{d}y}{\mathrm{d}x}=2(2k+5)x+k$	M1	attempt to differentiate, equate to zero and substitute x value back in to obtain a y value
	When $\frac{\mathrm{d}y}{\mathrm{d}x}=0,\ x=\frac{-k}{2(2k+5)},\ y=\frac{8k+20-k^2}{4(2k+5)}$	M1	consider $y=0$ in order to obtain critical values
	When $y=0$, obtain critical values of 10 and –2	A1	correct critical values
	$-2<k<10$	A1	correct range
	Alternative 2:		
	$y=(2k+5)\left(\left(x+\frac{k}{2(2k+5)}\right)^2-\frac{k^2}{4(2k+5)}\right)+1$	M1	attempt to complete the square and consider '$1-\frac{k^2}{4(2k+5)}$'
	Looking at $1-\frac{k^2}{4(2k+5)}=0$ leads to	M1	attempt to solve above = to 0, to obtain critical values
	critical values of 10 and –2	A1	correct critical values
	$-2<k<10$	A1	correct range

2	$\dfrac{\tan\theta+\cot\theta}{\operatorname{cosec}\theta}=\dfrac{\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}}{\frac{1}{\sin\theta}}$	M1	for $\tan\theta=\dfrac{\sin\theta}{\cos\theta}, \cot\theta=\dfrac{\cos\theta}{\sin\theta}$ and $\operatorname{cosec}\theta=\dfrac{1}{\sin\theta}$; allow when used
	$=\dfrac{\frac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}}{\frac{1}{\sin\theta}}$	M1	dealing correctly with fractions in the numerator; allow when seen
	$=\dfrac{1}{\cos\theta}$	M1	use of the appropriate identity; allow when seen
	$=\sec\theta$	A1	must be convinced it is from completely correct work (beware missing brackets)
	Alternative:		
	$\dfrac{\tan\theta+\cot\theta}{\operatorname{cosec}\theta}=\dfrac{\frac{\tan^2\theta+1}{\tan\theta}}{\operatorname{cosec}\theta}$	M1	for either $\tan\theta=\dfrac{1}{\cot\theta}$ or $\cot\theta=\dfrac{1}{\tan\theta}$ and $\operatorname{cosec}\theta=\dfrac{1}{\sin\theta}$; allow when used
	$=\dfrac{\sec^2\theta}{\tan\theta\frac{1}{\sin\theta}}$	M1	dealing correctly with fractions in numerator; allow when seen
	$=\dfrac{\sec^2\theta}{\sec\theta}$	M1	use of the appropriate identity; allow when seen
	$=\sec\theta$	A1	must be convinced it is from completely correct work
3	$\mathbf{A}^{-1}=\dfrac{1}{2}\begin{pmatrix}3 & -2\\ -5 & 4\end{pmatrix}$	B1	$\dfrac{1}{2}$ multiplied by a matrix
		B1	for matrix
	$\begin{pmatrix}x\\ y\end{pmatrix}=\dfrac{1}{2}\begin{pmatrix}3 & -2\\ -5 & 4\end{pmatrix}\begin{pmatrix}8\\ 9\end{pmatrix}$	M1	attempt to use the inverse matrix, must be pre-multiplication
	$\begin{pmatrix}x\\ y\end{pmatrix}=\dfrac{1}{2}\begin{pmatrix}6\\ -4\end{pmatrix}$		
	$x=3, y=-2$	A1, A1	

4 (i)	Area = $\left(\frac{1}{2}\times 12^2\times 1.7\right)+\left(\frac{1}{2}\times 12^2\sin(2\pi-1.7-2.4)\right)$	B1,B1	B1 for sector area, allow unsimplified B1 for correct angle *BOC*, allow unsimplified
		M1	correct attempt at area of triangle, allow unsimplified using *their* angle *BOC* (Their angle *BOC* must not be 1.7 or 2.4)
	= awrt 181	A1	
(ii)	$BC^2 = 12^2+12^2-(2\times 12\times 12\cos 2.1832)$ or $BC = 2\times 12\times\sin\left(\frac{2\pi-4.1}{2}\right)$	M1	correct attempt at *BC*, may be seen in **(i)**, allow if used in **(ii)**. Allow use of *their* angle *BOC*.
	$BC = 21.296$	A1	
	Perimeter = $(12\times 1.7)+12+12+21.296$	B1 M1	for arc length, allow unsimplified for a correct 'plan' (an arc + 2 radii and *BC*)
	= 65.7	A1	
5 (a) (i)	20160	B1	
(ii)	$3\times {}^6P_4\times 2$ = 2160	B1,B1	B1 for ${}^6P_4$ (must be seen in a product) B1 for all correct, with no further working
(iii)	$5\times 2\times {}^6P_4$ = 3600	B1,B1 B1	B1 for ${}^6P_4$ (must be seen in a product) B1 for 5 (must be in a product) B1 for all correct, with no further working
	Alternative 1: ${}^6C_4\times 5!\times 2$ = 3600	B2 B1	for ${}^6C_4\times 5!$ for ${}^6C_4\times 5!\ \times 2$
	Alternative 2: $\left({}^7P_5-{}^6P_5\right)\times 2$ = 3600	B2 B1	for $\left({}^7P_5-{}^6P_5\right)$ for $\left({}^7P_5-{}^6P_5\right)\times 2$
	Alternative 3: $2!\left({}^6P_4+\left({}^6P_1\times{}^5P_3\right)+\left({}^6P_2\times{}^4P_2\right)+\left({}^6P_3\times{}^3P_1\right)+{}^6P_4\right)$ = 3600	B2 B1	4 terms correct or omission of 2! in each term all correct

(b) (i)	${}^{14}C_4 \times {}^{10}C_4$ or ${}^{14}C_8 \times {}^{8}C_4$ (or numerical or factorial equivalent) $= 210210$	B1,B1	B1 for either ${}^{14}C_4$ or ${}^{14}C_8$ as part of a product B1 for correct answer, with no further working
(ii)	${}^{8}C_4 \times {}^{6}C_4$ $= 1050$	B1,B1	B1 for either ${}^{8}C_4$ or ${}^{6}C_4$ as part of a product B1 for correct answer with no further working
6 (i)	10ln4 or 13.9 or better	B1	
(ii)	$\left(\frac{\mathrm{d}x}{\mathrm{d}t}=\right)\frac{20t}{t^2+4}-4$	M1 B1	attempt to differentiate and equate to zero $\frac{20t}{t^2+4}$ or equivalent seen
	When $\frac{\mathrm{d}x}{\mathrm{d}t}=0, \frac{20t}{t^2+4}=4$	DM1	attempt to solve *their* $\frac{\mathrm{d}x}{\mathrm{d}t}=0$, must be a 2 or 3 term quadratic equation with real roots
	leading to $t^2-5t+4=0$ $t=1, t=4$	A1	for both

(iii)	If $(v =)\ \dfrac{20t}{t^2+4} - 4$		
	$(a =)\ \dfrac{20(t^2+4) - 20t(2t)}{(t^2+4)^2}$	M1 A1 A1	attempt to differentiate *their* $\dfrac{dx}{dt}$ $20(t^2+4)$ $20t(2t)$
	$20(4-t^2)$ or $80-20t^2$ or $4-t^2$ or equivalent expression involving $-t^2$	A1	$20(4-t^2)$ or $80-20t^2$ or $4-t^2$
	When acceleration is 0, $t = 2$ only	B1	$t = 2$, dependent on obtaining first and second A marks
	Alternative 1 for first 3 marks: If$(v =)\ \dfrac{20t-4t^2-16}{t^2+4}$ $(a =)\dfrac{(t^2+4)\ (20-8t) - (20t-4t^2-16)\ (2t)}{(t^2+4)^2}$	M1 A1 A1	attempt to differentiate *their* $\dfrac{dx}{dt}$ for $(t^2+4)(20-8t)$ for $(20t-4t^2-16)(2t)$
	Alternative 2 for M1 mark: If $(v =)\ 20t(t^2+4)^{-1} - 4$ $(a =)20t\left(-2t(t^2+4)^{-2}\right) + 20(t^2+4)^{-1}$	M1	attempt to differentiate *their* $\dfrac{dx}{dt}$
	Alternative 3 for the first 3 marks If $(v =)\ (20t-4t^2-16)(t^2+4)^{-1}$ $(a =)(20t-4t^2-16)\left(-2t(t^2+4)^{-2}\right) + (20-8t)(t^2+4)^{-1}$ Numerator $= -2t(20t-4t^2-16) + (20-8t)\ (t^2+4)$	M1 A1 A1	attempt to differentiate *their* $\dfrac{dx}{dt}$ for $2t(20t-4t^2-15)$ for $(20-8t)(t^2+4)$
7 (i)	$\overrightarrow{DA} = 3\mathbf{a} - \mathbf{b}$	B1	mark final answer, allow unsimplified
(ii)	$\overrightarrow{DB} = 7\mathbf{a} - \mathbf{b}$	B1	mark final answer, allow unsimplified
(iii)	$\overrightarrow{AX} = \lambda(4\mathbf{a} + \mathbf{b})$	B1	mark final answer, allow unsimplified
(iv)	$\overrightarrow{DX} = 3\mathbf{a} - \mathbf{b} + \lambda(4\mathbf{a} + \mathbf{b})$	M1 A1	*their* **(i)** + *their* **(iii)** or equivalent valid method or $3\mathbf{a} - \mathbf{b}$ + *their* **(iii)** Allow unsimplified

(v)	$3\mathbf{a}-\mathbf{b}+\lambda(4\mathbf{a}+\mathbf{b})=\mu(7\mathbf{a}-\mathbf{b})$ Equate like vectors: $3+4\lambda=7\mu$ $-1+\lambda=-\mu$ leads to $\lambda=\frac{4}{11}$, $\mu=\frac{7}{11}$	M1 DM1 A1,A1	equating *their* **(iv)** and $\mu\times$ *their* **(ii)** for an attempt to equate like vectors and attempt to solve 2 linear equations for λ and μ A1 for each
8 (i)	$5e^{2x}-\frac{1}{2}e^{-2k}\quad(+c)$	B1, B1	B1 for each term, allow unsimplified
(ii)	$\left(5e^{2k}-\frac{1}{2}e^{-2k}\right)-\left(5e^{-2k}-\frac{1}{2}e^{2k}\right)$	M1 A1	use of limits provided integration has taken place. Signs must be correct if brackets are not included. allow any correct form
(iii)	$\left(5e^{2k}-\frac{1}{2}e^{-2k}\right)\left(5e^{-2k}-\frac{1}{2}e^{2k}\right)=-60$ or $\frac{11}{2}e^{2k}-\frac{11}{2}e^{-2k}=-60$ or equivalent leading to $11e^{2k}-11e^{-2k}+120=0$	B1 DB1	correct expression from **(ii)** either simplified or unsimplified equated to – 60 , must be first line seen. must be convinced as **AG**
(iv)	$11y^2+120y-11=0$ $(11y-1)(y+11)=0$ leading to $k=\frac{1}{2}\ln\frac{1}{11}$, $\ln\frac{1}{\sqrt{11}}$, $-\ln\sqrt{11}$, $-\frac{1}{2}\ln 11$	M1 DM1 A1	attempt to obtain a quadratic equation in *y* or e^{2k} and solve to get *y* or e^{2k} (only need positive solution) attempt to deal with e to get $k=$. any of given answers only.

9	$\frac{dy}{dx} = 4 - 6\sin 2x$	M1,A1	M1 for attempt to differentiate A1 for all correct
	When $x = \frac{\pi}{4},\ y = \pi$	B1	for *y*
	$\frac{dy}{dx} = -2$ so gradient of normal = $\frac{1}{2}$	DM1	for substitution of $x = \frac{\pi}{4}$ into *their* $\frac{dy}{dx}$ and use of '$m_1m_2 = -1$', dependent on first M1
	Normal equation $y - \pi = \frac{1}{2}\left(x - \frac{\pi}{4}\right)$	DM1	correct attempt to obtain the equation of the normal, dependent on previous DM mark
	When $x = 0,\ y = \frac{7\pi}{8}$	A1	must be terms of π
	When $y = 0, x = -\frac{7\pi}{4}$	A1	must be terms of π
	Area = $\frac{1}{2} \times \frac{7\pi}{4} \times \frac{7\pi}{8} \quad = \frac{49\pi^2}{64}$	B1ft	Follow through on *their x* and *y* intercepts; must be exact values
10 (a)	$\cos^2 3x = \frac{1}{2}, \qquad \cos 3x = (\pm)\frac{1}{\sqrt{2}}$ $3x = 45°, 135°, 225°, 315°$ $x = 15°, 45°, 75°, 105°$	M1 A1,A1	complete correct method, dealing with sec and 3, correctly A1 for each correct pair
(b)	$3(\cot^2 y + 1) + 5\cot y - 5 = 0$ Leading to $3\cot^2 y + 5\cot y - 2 = 0$ or	M1	use of a correct identity to get an equation in terms of one trig ratio only
	$2\tan^2 y - 5\tan y - 3 = 0$ $(3\cot y - 1)(\cot y + 2) = 0$ or $(\tan y - 3)(2\tan y + 1) = 0$	M1	for $\cot y = \frac{1}{\tan y}$ to obtain either a quadratic equation in $\tan y$ or solutions in terms of $\tan y$; allow where appropriate
	$\tan y = 3, \qquad \tan y = \frac{1}{2}$	M1	for solution of a quadratic equation in terms of either $\tan y$ or $\cot y$
	$y = 71.6°, 251.6° \qquad 153.4°, 333.4°$	A1,A1	A1 for each correct 'pair'
(c)	$\sin\left(z + \frac{\pi}{3}\right) = \frac{1}{2}$	M1	completely correct method of solution
	$z + \frac{\pi}{3} = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}$	A1	one correct solution in range
	$z = \frac{\pi}{2}, \frac{11\pi}{6}$ (allow 1.57, 5.76)	M1 A1	correct attempt to obtain a second solution within the range second correct solution (and no other)