**The City School  
North Nazimabad Boys Campus**

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**Explanation of Logarithms**

Introduction to Logarithms

In its simplest form, a logarithm answers the question:

**How many of *one number* do we multiply to get *another number?***

Example: How many 2s do we multiply to get 8?

Answer: **2 × 2 × 2 = 8**, so we needed to multiply 3 of the **2**s to get **8**

So the logarithm is 3

How to Write it

We write "the number of 2s we need to multiply to get 8 is 3" as:

log2(8) = 3

So these two things are the same:

|  |
| --- |
| logarithm concept |

The number we are multiplying is called the "base", so we can say:

* "the logarithm of 8 with base 2 is 3"
* or "log base 2 of 8 is 3"
* or "the base-2 log of 8 is 3"

Notice we are dealing with three numbers:

* the **base**: the number we are multiplying (a "2" in the example above)
* how many times to use it in a multiplication (3 times, which is the **logarithm**)
* The number we want to get (an "8")

More Examples

Example: What is log5(625) ... ?

We are asking "how many 5s need to be multiplied together to get 625?"

**5 × 5 × 5 × 5 = 625**, so we need 4 of the 5s

Answer: log5(625) = 4

Example: What is log2(64) ... ?

We are asking "how many 2s need to be multiplied together to get 64?"

**2 × 2 × 2 × 2 × 2 × 2 = 64**, so we need 6 of the 2s

Answer: log2 (64) = 6

Exponents

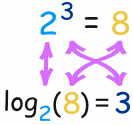
Exponents and Logarithms are related, let's find out how ...

|  |  |
| --- | --- |
| 2 cubed | The **exponent** says **how many times**to use the number in a multiplication.  In this example: **23 = 2 × 2 × 2 = 8**  *(2 is used 3 times in a multiplication to get 8)* |

So a logarithm answers a question like this:

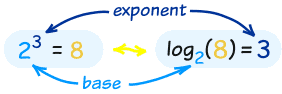
Logarithm Question

In this way:



**The logarithm tells us what the exponent is!**

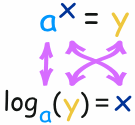
In that example the "base" is 2 and the "exponent" is 3:



So the logarithm answers the question:

**What exponent do we need**  *(for one number to become another number)***?**

The **general** case is:



Example: What is log10(100) ... ?

102 = 100

So an exponent of 2 is needed to make 10 into 100, and:

log10(100) = 2

Example: What is log3(81) ... ?

34 = 81

So an exponent of 4 is needed to make 3 into 81, and:

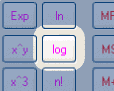
log3(81) = 4

Common Logarithms: Base 10

Sometimes a logarithm is written **without** a base, like this:

log(100)

This ***usually*** means that the base is really 10.



It is called a "common logarithm". Engineers love to use it.

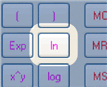
On a calculator it is the "log" button.

It is how many times we need to use 10 in a multiplication, to get our desired number.

Example: **log(1000) = log10(1000) = 3**

 Natural Logarithms: Base "e"

Another base that is often used is [e (Euler's Number)](https://www.mathsisfun.com/numbers/e-eulers-number.html) which is about 2.71828.



This is called a "natural logarithm". Mathematicians use this one a lot.On a calculator it is the "ln" button.It is how many times we need to use "e" in a multiplication, to get our desired number.

Example: **ln(7.389) = loge(7.389) ≈ 2**

Because **2.718282 ≈ 7.389**  
But Sometimes There Is Confusion ... !

Mathematicians use "log" (instead of "ln") to mean the natural logarithm. This can lead to confusion:

|  |  |  |  |
| --- | --- | --- | --- |
| **Example** | **Engineer Thinks** | **Mathematician Thinks** |  |
| log(50) | log10(50) | loge(50) | confusion |
| ln(50) | loge(50) | loge(50) | no confusion |
| log10(50) | log10(50) | log10(50) | no confusion |

So, be careful when you read "log" that you know what base they mean!

Logarithms Can Have Decimals

All of our examples have used whole number logarithms (like 2 or 3), but logarithms can have decimal values like 2.5, or 6.081, etc.

Example: what is **log10(26) ... ?**

|  |  |
| --- | --- |
| log | Get your calculator, type in **26** and press **log**  Answer is: **1.41497...** |

The logarithm is saying that 101.41497... = 26   
(10 with an exponent of **1.41497...** equals 26)

|  |  |  |
| --- | --- | --- |
| This is what it looks like on a graph:  See how nice and smooth the line is. |  | https://www.mathsisfun.com/algebra/images/log-10-26.gif |

Read [Logarithms Can Have Decimals](https://www.mathsisfun.com/algebra/logarithms-decimals.html) to find out more.

Negative Logarithms

|  |  |
| --- | --- |
|  | Negative? But logarithms deal with multiplying.  What could be the opposite of multiplying? **Dividing!** |

A negative logarithm means how many times**to divide** by the number.

We could have just one divide:

Example: What is log8(0.125) ... ?

Well, 1 ÷ 8 = 0.125,

So log8(0.125) = −1

Or many divides:

Example: What is log5(0.008) ... ?

**1 ÷ 5 ÷ 5 ÷ 5** = **5−3**,

So log5(0.008) = −3

It All Makes Sense

Multiplying and Dividing are all part of the same simple pattern.

Let us look at some Base-10 logarithms as an example:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **Number** | **How Many 10s** | **Base-10 Logarithm** | |
| larger-smaller | .. etc.. |  |  |  |
| 1000 | 1 × 10 × 10 × 10 | log10(1000) | = 3 |
| 100 | 1 × 10 × 10 | log10(100) | = 2 |
| 10 | 1 × 10 | log10(10) | = 1 |
| 1 | 1 | log10(1) | = 0 |
| 0.1 | 1 ÷ 10 | log10(0.1) | = −1 |
| 0.01 | 1 ÷ 10 ÷ 10 | log10(0.01) | = −2 |
| 0.001 | 1 ÷ 10 ÷ 10 ÷ 10 | log10(0.001) | = −3 |
| .. etc.. |  |  |  |

Looking at that table, see how positive, zero or negative logarithms are really part of the same (fairly simple) pattern.