

**ADDITIONAL ATHEMATICS
FORM 5
MODULE 3**

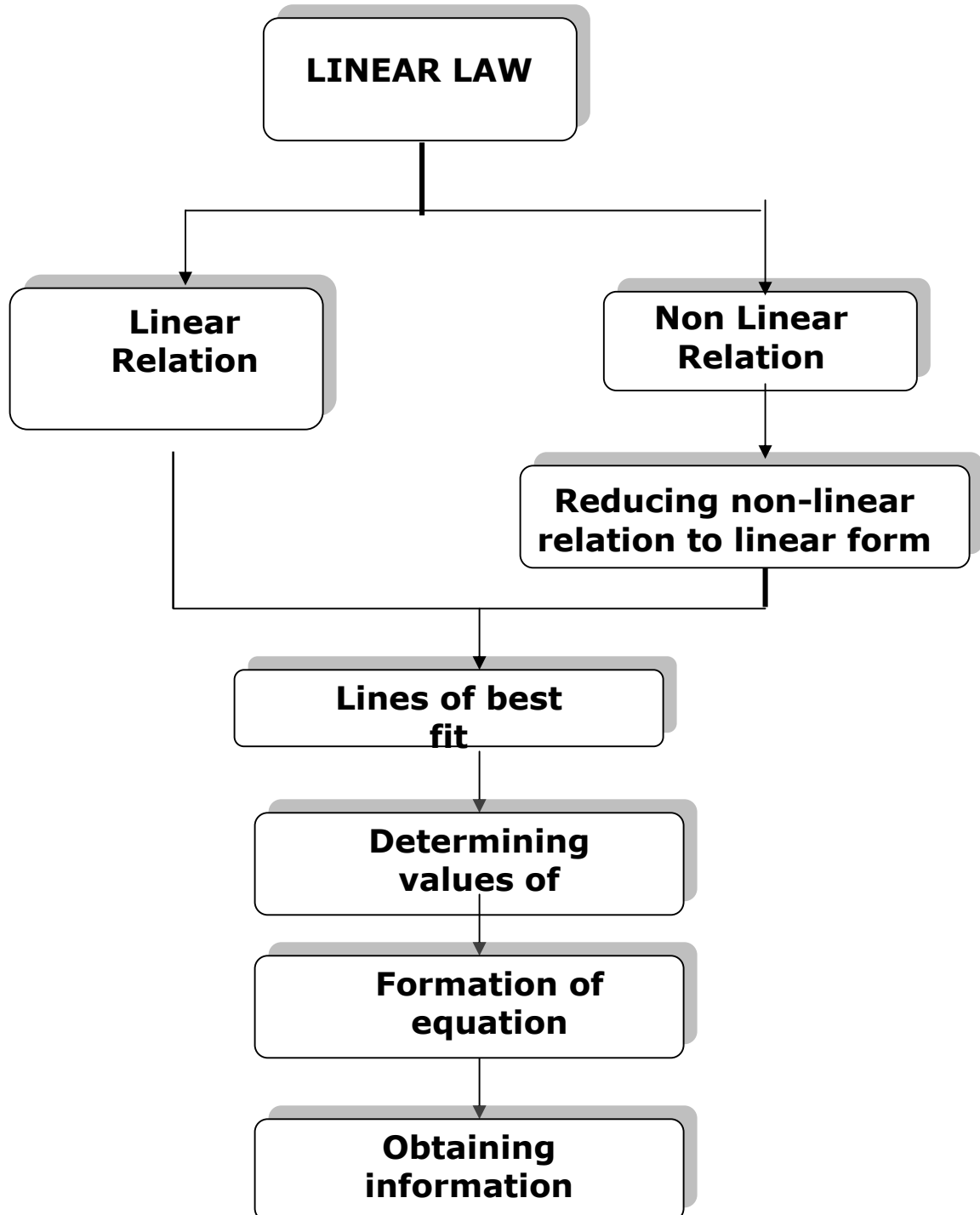
LINEAR LAW

CHAPTER 2 : LINEAR LAW

	Page
Contents	
3.0 CONCEPT MAP	2
3.1 UNDERSTAND AND USE THE CONCEPT OF LINES OF BEST FIT	3
3.1.1 Draw lines of best fit by inspection of given data Exercises 1	3
3.1.2 Write equations for lines of best fit Exercises 2	4
3.1.3 Determine values of variables from a) lines of best fit b) equations of lines of best fit Exercises 3	5
	6
3.2 APPLY LINEAR LAW TO NON- LINEAR RELATIONS	7
3.2.1 Reduce non-linear relations to linear form Exercises 4	7
3.2.2 Determine values of constants of non-lineas relations given Exercises 5	8
3.2.3 Steps to plot a straight line Exercises 6	10
3.3 SPM Questions	17
3.4 Assessment test	27
3.5 Answers	30

CONCEPT MAP

CHAPTER 2 : LINEAR LAW



3.1 UNDERSTAND AND USE THE CONCEPT OF LINES OF BEST FIT

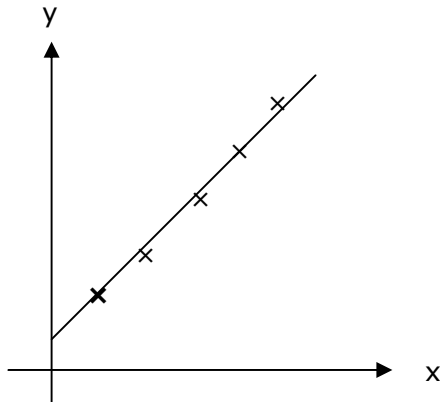
THE CONCEPT OF LINES OF BEST FIT.

The properties of the line of best fit.

- The straight line is drawn such away that it passes through as many points as possible.
- The number of points that do not lie on the straight line drawn should be more or less the same both sides of the straight line.

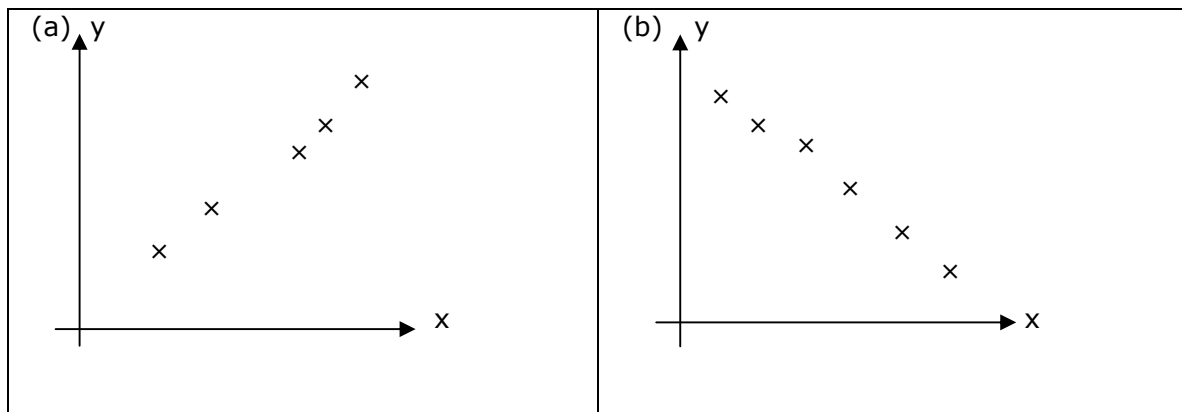
Example.

Draw the line of best fit for the given graph below.



1. Draw the line of best fit for the following diagrams.

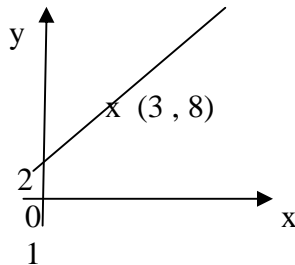
Exercises 1



3.1.2 Write equations for lines of best fit

Exercises 2

Example 1



The equation that can be formed is:

$$Y = mX + c$$

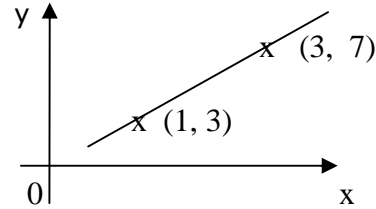
$$\text{Gradient, } m = \frac{8-2}{3-0} = 2$$

$$Y\text{-intercept} = 2$$

$$\text{Therefore, } y = 2x + 2$$

Linear form

Example 2



The equation that can be formed is:

$$Y = mX + c$$

$$\text{Gradient, } m = \frac{7-3}{3-1} = 2$$

The straight line also passes through the

Point (1, 3)

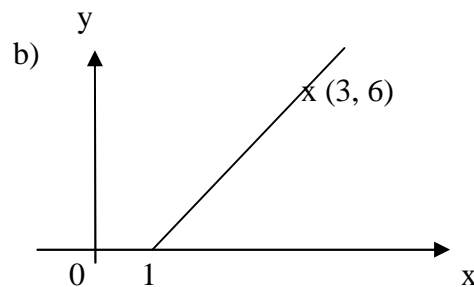
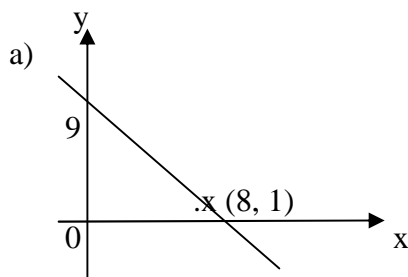
Another equation can be formed is,

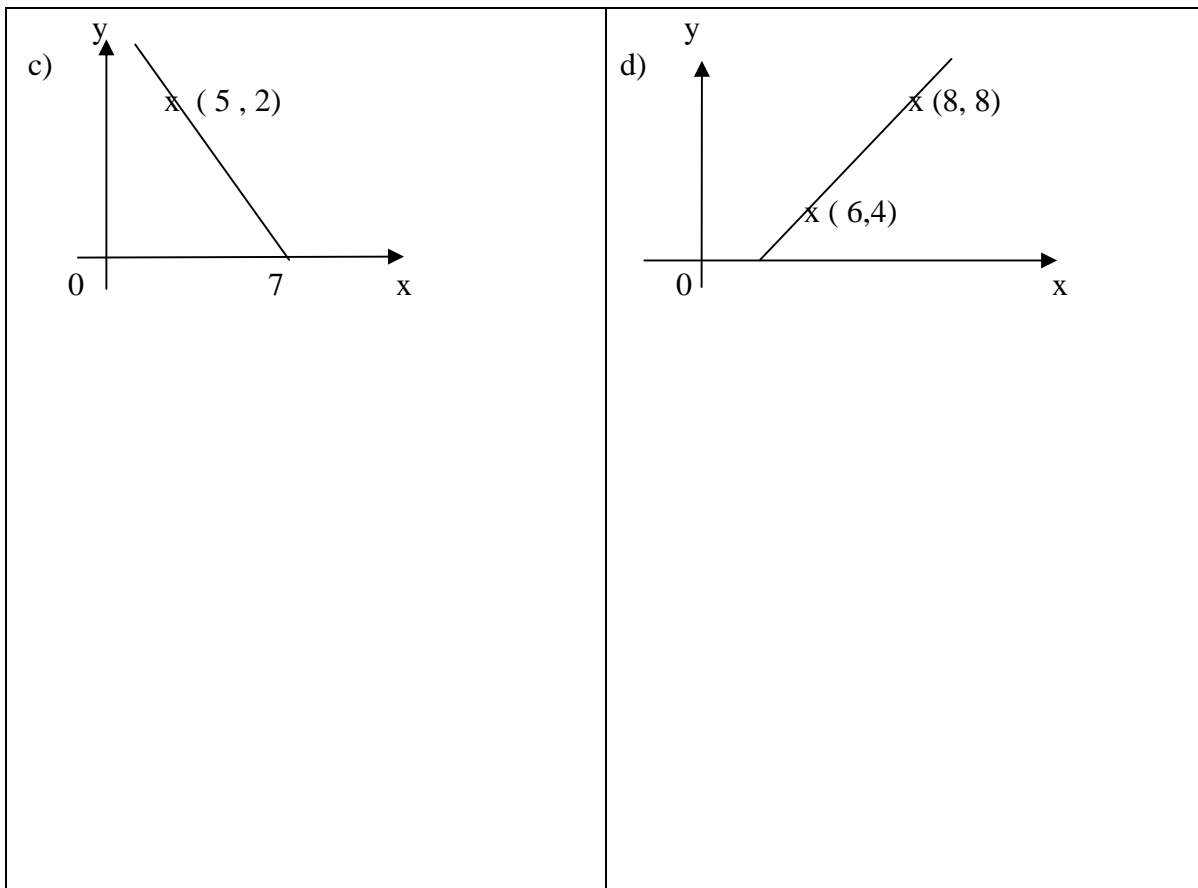
$$3 = 2(1) + c$$

$$.c = 1$$

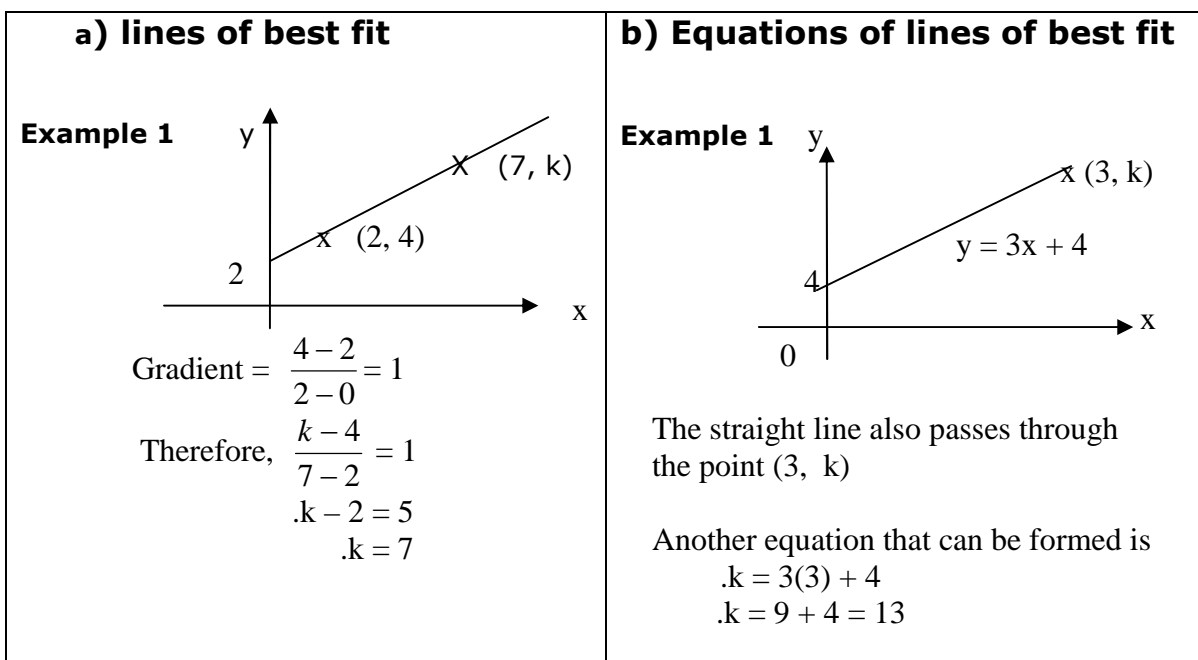
$$\text{Therefore, } y = 2x + 1$$

Linear form

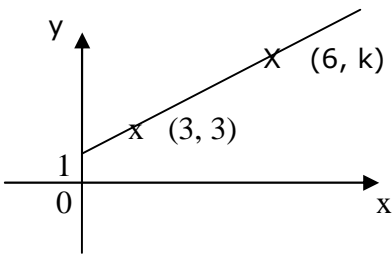
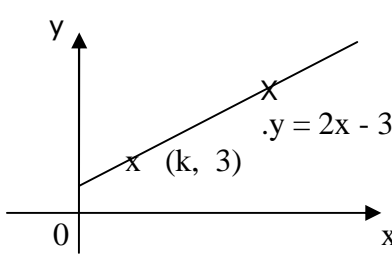
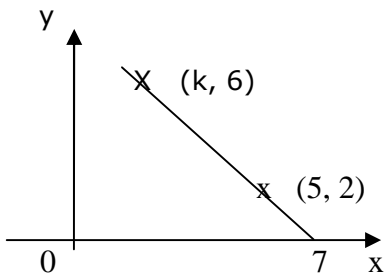
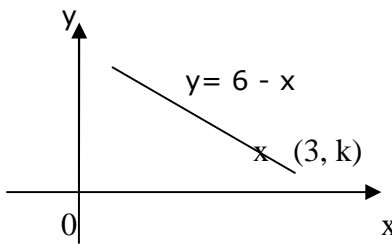
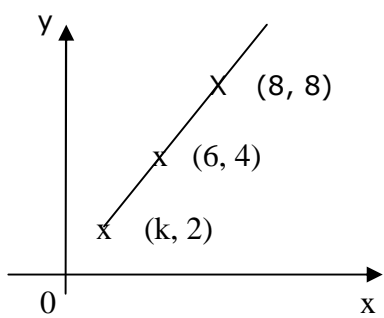
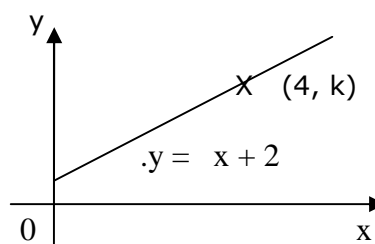




3.1.3 Determine values of variables from:



Exercises 3

a) lines of best fit	b) Equations of lines of best fit
<p>i)</p> 	<p>i)</p> 
<p>ii)</p> 	<p>ii)</p> 
<p>iii)</p> 	<p>iii)</p> 

3.2 APPLY LINEAR LAW TO NON- LINEAR RELATIONS

3.2.1 Reduce non- linear relations to linear form

$$Y = mX + c$$

Example 1

$$y = ax + \frac{x^2}{b}$$

Non- linear

$$\frac{y}{x} = \frac{ax}{x} + \frac{x^2}{xb}$$

Divided by x

$$\frac{y}{x} = a + x \frac{1}{b}$$

Linear form

$$Y = c + Xm$$

Example 1

a) $y = px^q$ ← Non- linear

$$\log_{10} y = \log_{10} (px^q)$$

Take log of Both sides

$$\log_{10} y = \log_{10} p + \log_{10} x^q$$

$$\log_{10} y = \log_{10} p + q \log_{10} x$$

Linear form

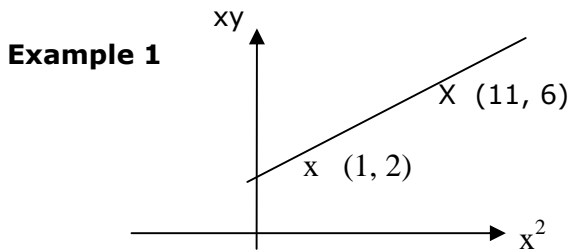
$$Y = c + mX$$

Exercises 4

Equation	Linear form	Y	X	m	c
a) $y^2 = ax + b$					
b) $y = ax^2 + bx$					
c) $\frac{a}{y} = \frac{b}{x} + 1$					
d) $y^2 = 5x^2 + 3x$					
e) $y = 3\sqrt{x} + \frac{5}{\sqrt{x}}$					
f) $y = ab^x$					
g) $y = \frac{4}{a^2}(x+b)^2$					

3.2.4 Determine values of constants of non-linear relations given

Exercises 5



The above figure shows part of a straight line graph drawn to represent the equation

$$. xy = ax^2 + b$$

Find the value of a and b

$$\text{Gradient, } a = \frac{6-2}{11-1}$$

$$A = \frac{2}{5}$$

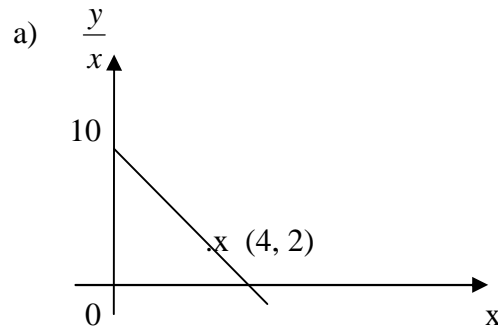
$$\text{Therefore, } xy = \frac{2}{5} x^2 + b$$

Another equation that can be formed is

$$.(1)(2) = \frac{2}{5} (1)^2 + b$$

$$b, = 2 - \frac{2}{5} = \frac{8}{5}$$

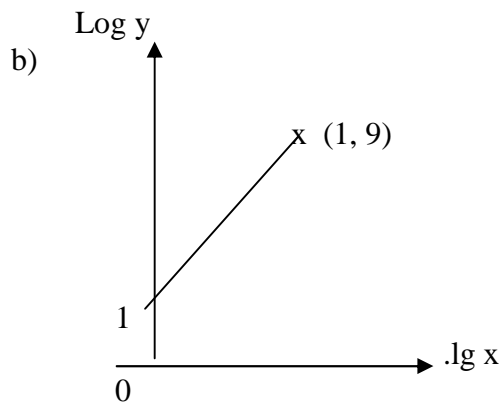
$$\text{Hence } a = \frac{2}{5}, \quad b = \frac{8}{5}$$



The above figure shows part of a straight line graph drawn to represent the equation

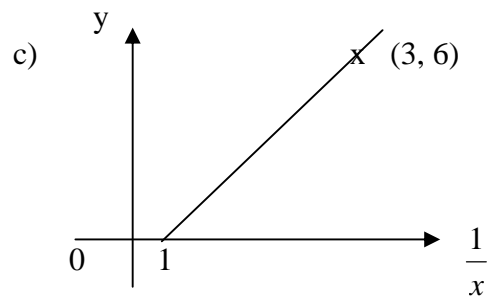
$$. y = ax^2 + bx$$

Find the value of a and b ,



The above figure shows part of a straight line graph drawn to represent the equation

of $y = ax^b$
Find the value of a and b ,

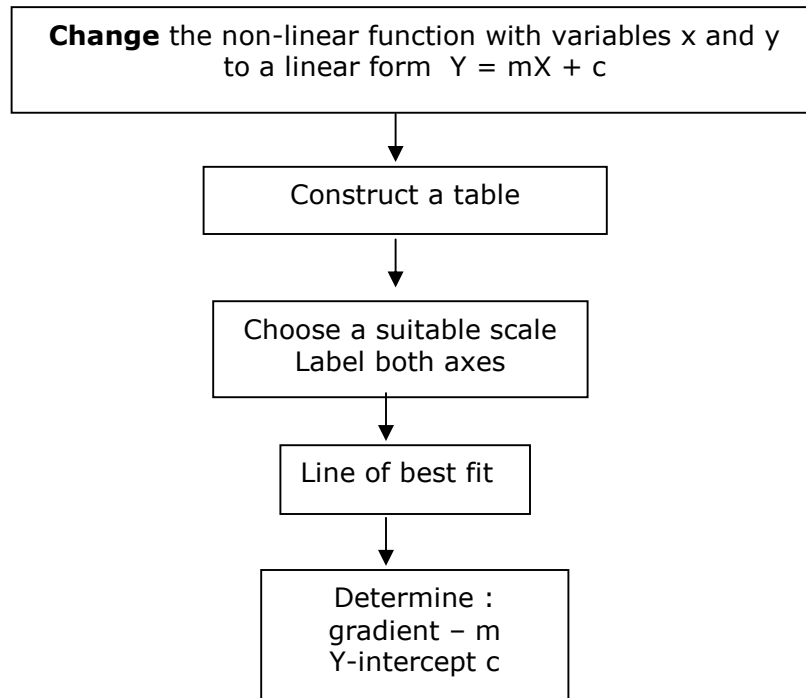


The above figure shows part of a straight line graph drawn to represent the equation

of $xy = a + bx$
Find the value of a and b ,

3.3 STEPS TO PLOT A STRAIGHT LINE

3.3.1. Using a graph paper.



QUESTIONS

X	2	3	4	5	6
Y	2	9	20	35	54

The above table shows the experimental values of two variables, x and y. It is known that x and y are related by the equation

$$y = px^2 + qx$$

Non- linear

a) Draw the line of best fit for $\frac{y}{x}$ against x

b) From your graph, find,
i) the initial velocity
ii) the acceleration

← Table

SOLUTION**STEP 1**

Reduce the non-linear
To the linear form

$$y = px^2 + qx$$

$$\frac{y}{x} = \frac{px^2}{x} + \frac{qx}{x}$$

The equation is divided throughout by x
To create a constant that is free from x
On the right-hand side i.e, q

$$\frac{y}{x} = px + q$$

Linear form
 $Y = mX + c$

$$\begin{array}{ccc} \uparrow & \uparrow \uparrow & \uparrow \\ Y & = mX & + c \end{array}$$

STEP 2

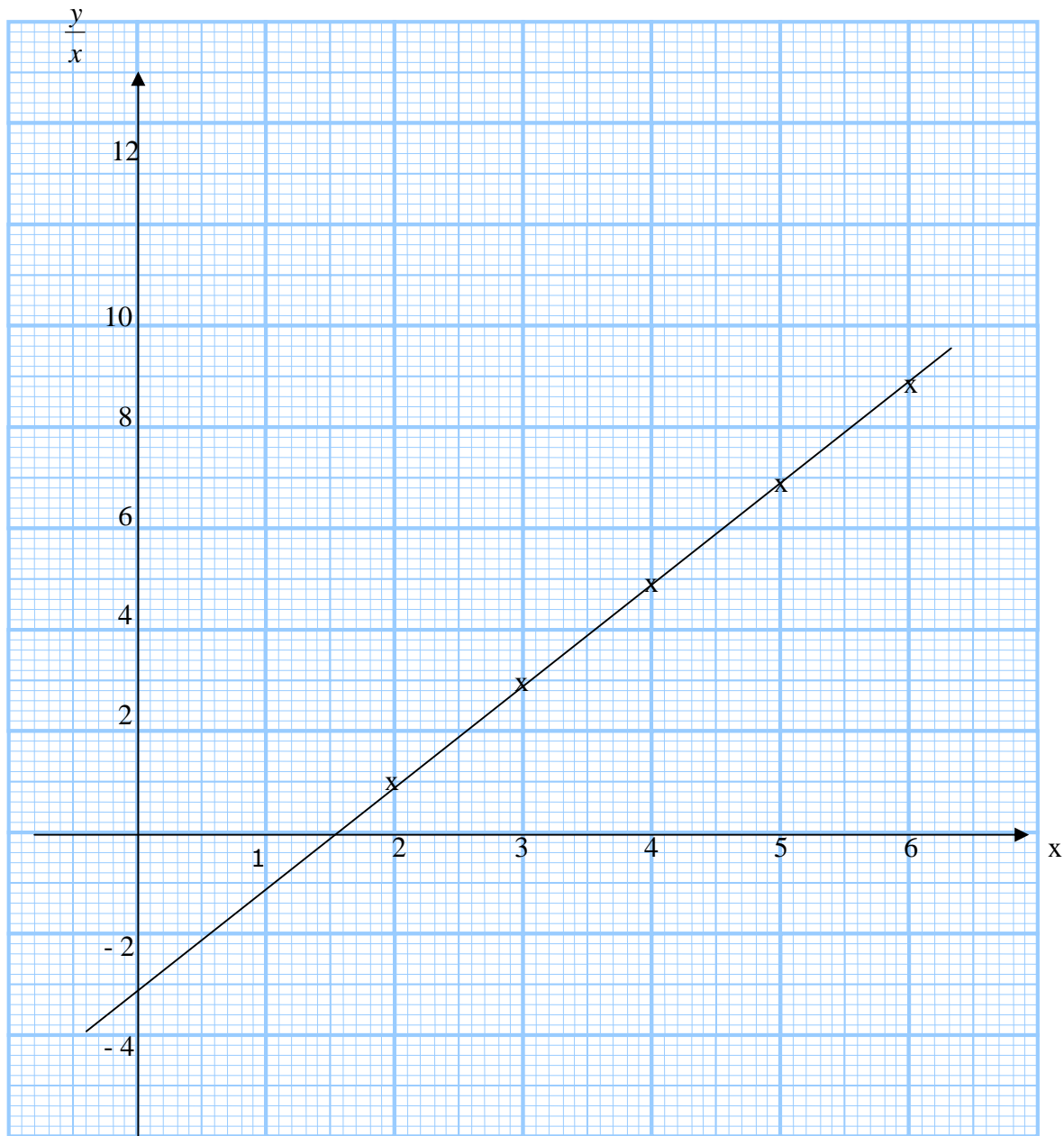
construct table

.x	2	3	4	5	6
.y	2	9	20	35	54
$\frac{y}{x}$	1	3	5	7	9

STEP 3

Using graph paper,

- Choose a suitable scale so that the graph drawn is as big as possible.
- Label both axis
- Plot the graph of Y against X and draw the line of best fit



STEP 4

From the graph,
find m and c

Gradient , $m = \frac{9-1}{6-2} = 2$

Construct a right-angled triangle,
So that two vertices are on the
line of best fit, calculate the
gradient, m

Y- intercept = $c = -3$

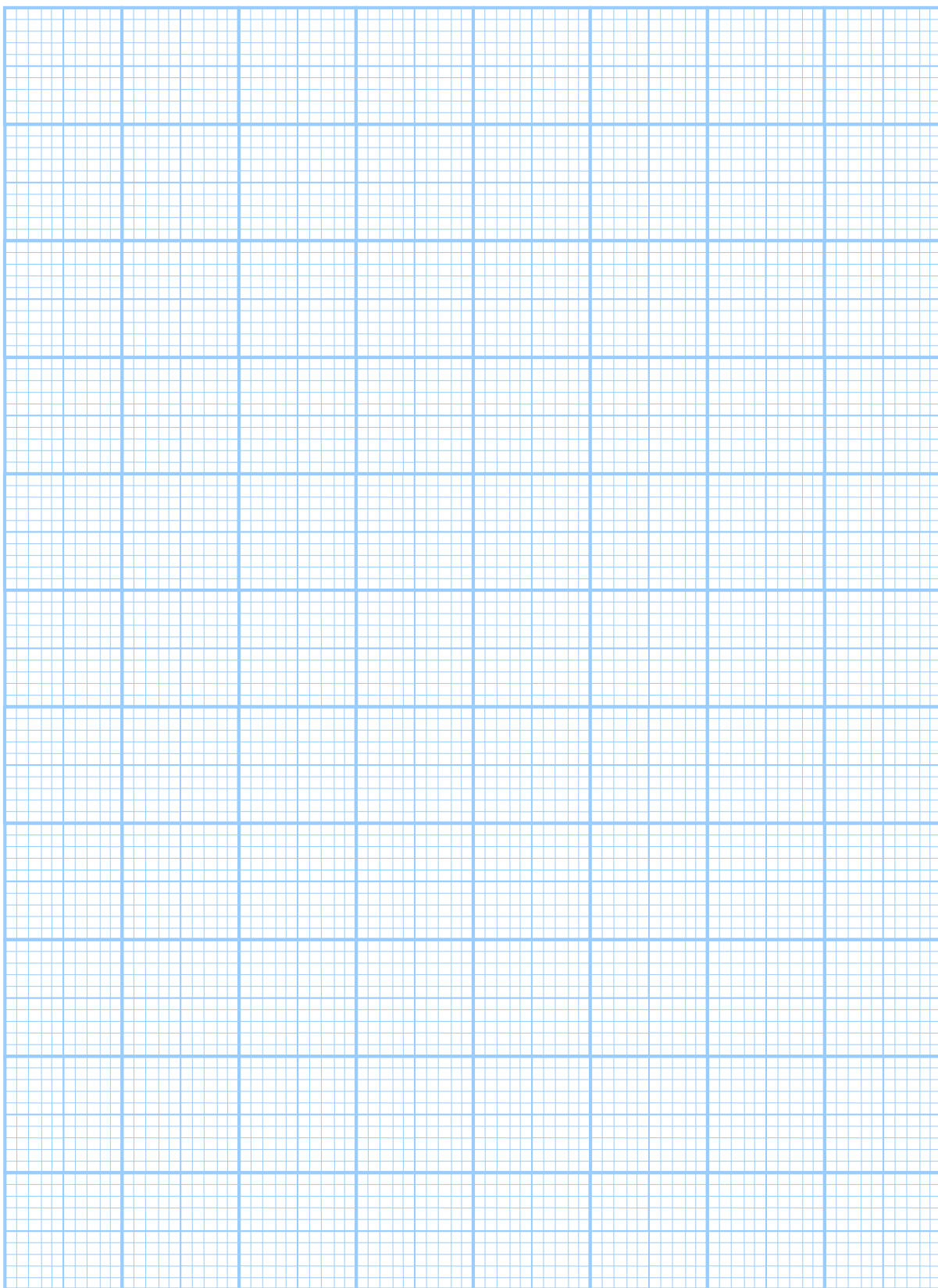
Determine the Y-intercept, c
.from the straight line graph

Exercises 6

1. The table below shows some experimental data of two related variable x and y
It is know that x and y are related by an equation in the form
 $y = ax + bx^2$, where. a and b are constants

x	1	2	3	4	5	6	7
y	7	16	24	24	16	0	-24

- a) Draw the straight line graph of $\frac{y}{x}$ against x
b) Hence, use the graph to find the values of a and b



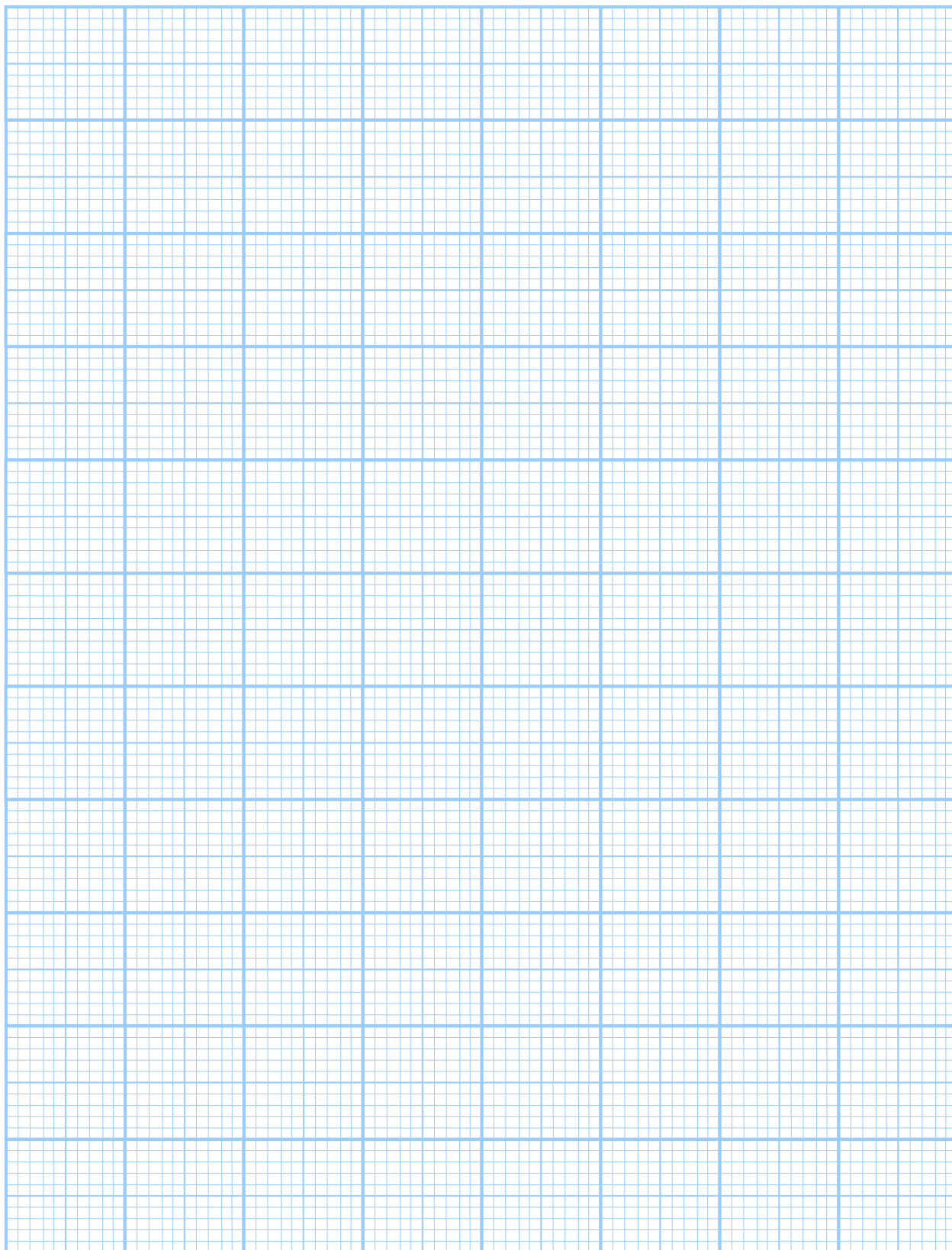
2. The table below shows some experimental data of two related variable x and y

x	0	2	4	6	8	10
y	1.67	1.9	2.21	2.41	2.65	2.79

It is known that x and y are related by an equation in the form

$$y = \frac{ax}{y} + \frac{b}{y}, \text{ where a and b are constants.}$$

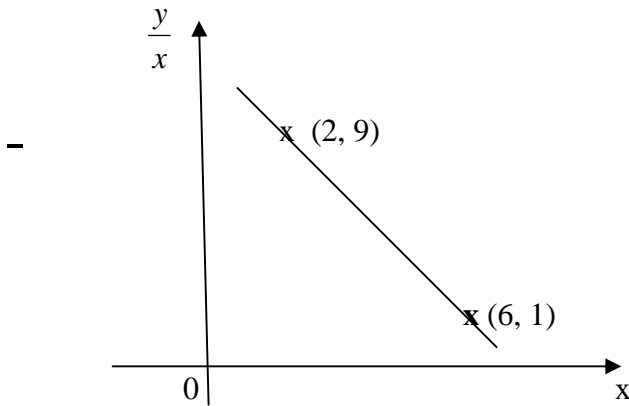
- a) Draw the straight line graph y^2 against x
- b) Hence, use the graph to find the values of a and b



4.0 SPM QUESTIONS

1. SPM 2003(paper 1, question no 10)

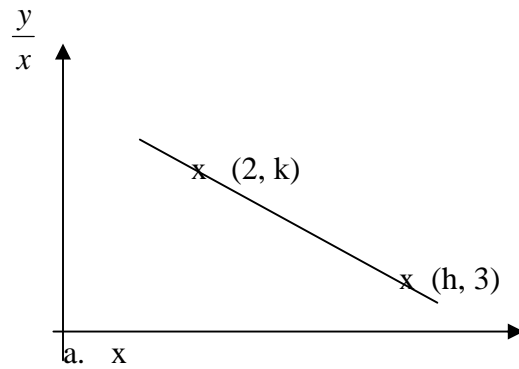
x and y are related by equation $y = px^2 + qx$, where p and q are constants. A straight line is obtained by plotting $\frac{y}{x}$ against x, as show in Diagram below



Calculate the values of p and q.

2. SPM 2004(paper 1, question no 13)

Diagram below shows a straight line graph of $\frac{y}{x}$ against x

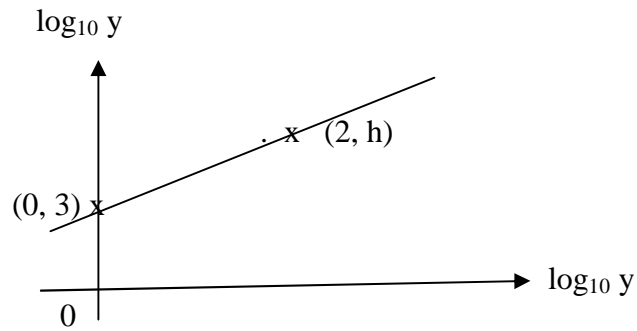


Given that $y = 6x - x^2$, calculate the value of k and of h

3. SPM 2005(paper 1, question no 13)

The variable x and y are related by the equation $y = kx^4$ where k is a constant.

- Convert the equation $y = kx^4$ to linear form
- Diagram below shows the straight line obtained by plotting $\log_{10} y$ against $\log_{10} x$



Find the value of

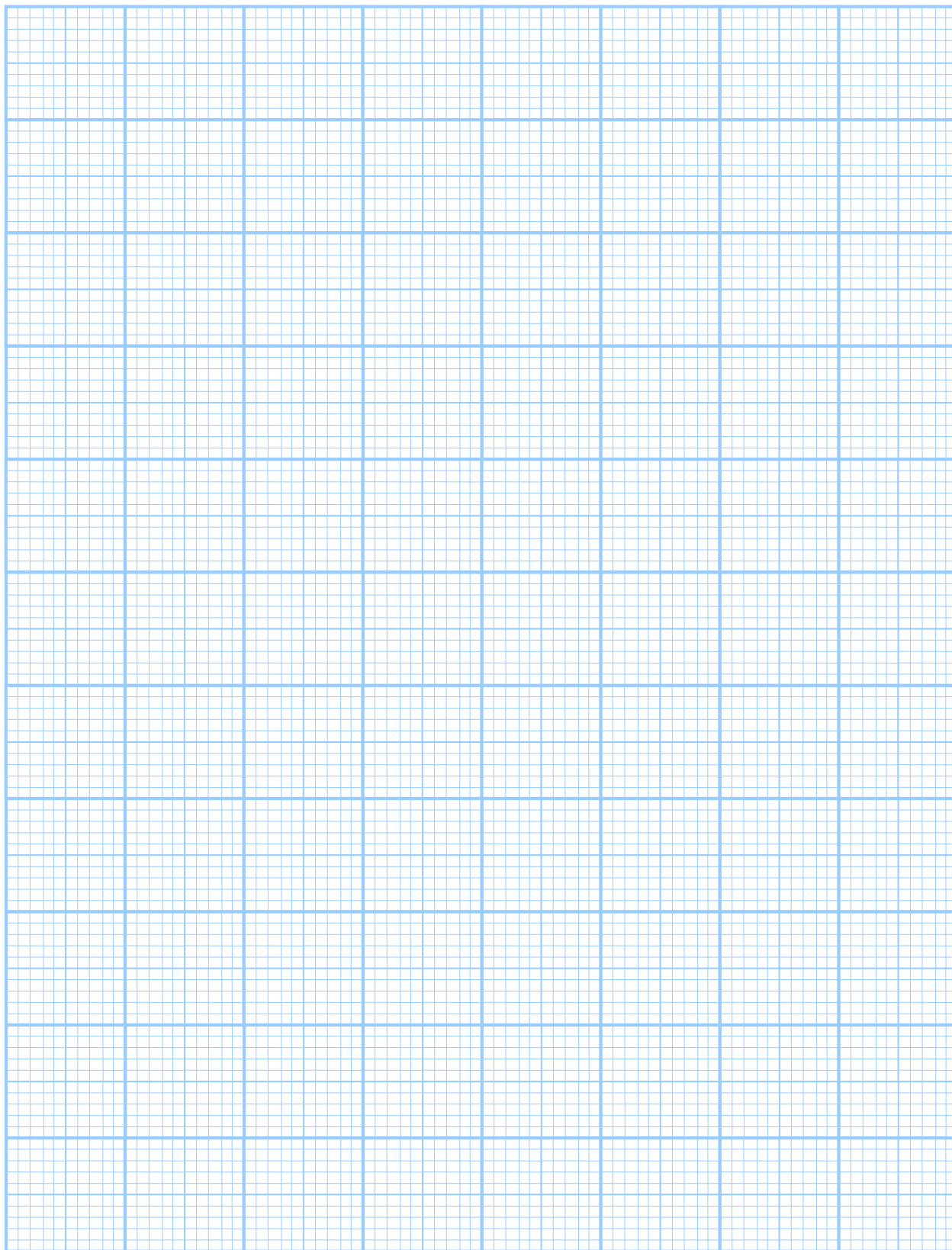
- $\log_{10} k$
- h .

4. SPM 2003(paper 2, question no 7)

Table 1 shows the values of two variables, x and y , obtained from an experiment. It is known that x and y are related by the equation $y = pk^{x^2}$, where p and k are constants.

x	1.5	2.0	2.5	3.0	3.5	4.0
y	1.59	1.86	2.40	3.17	4.36	6.76

- (a) Plot $\log y$ against x^2 .
Hence, draw the line of best fit
- (b) Use the graph in (a) to find value of
- (i) p
 - (ii) k



5. SPM 2004(paper 2, question no 7)

Table 1 shows the values of two variables, x and y , obtained from an experiment.

Variables x and y are related by the equation $y = pk^x$, where p and k are constants.

x	2	4	6	8	10	12
y	3.16	5.50	9.12	16.22	28.84	46.77

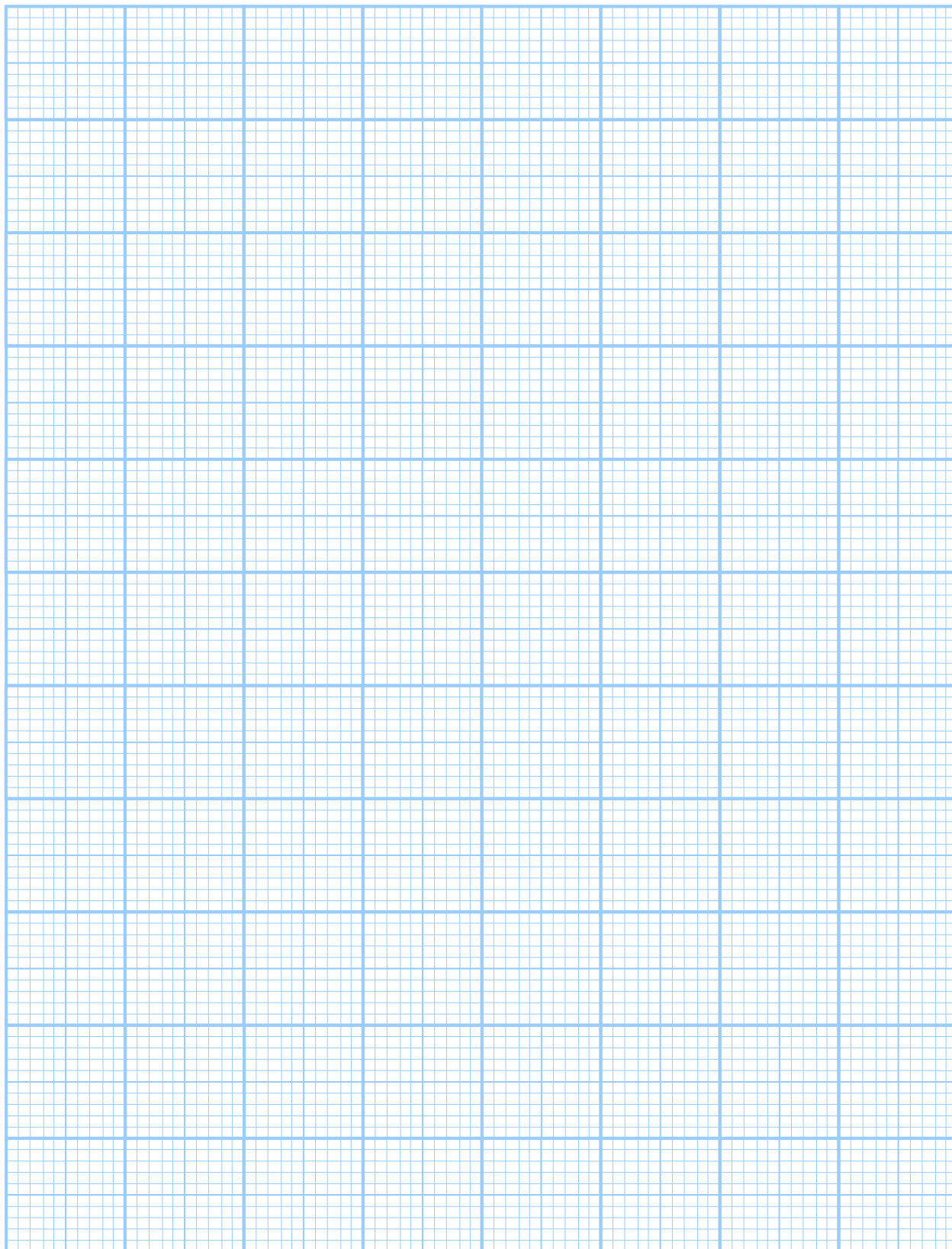
- (a) Plot $\log_{10} y$ against x by using scale of 2 cm to 2 units on the x -axis and 2 cm to 0.2 unit on the \log_{10} -axis.

Hence, draw the line of best fit

- (b) Use the graph in (a) to find value of

(i) p

(ii) k



6. SPM 2005(paper 2, question no 7)

Table 1 shows the values of two variables, x and y , obtained from an experiment.

The variables x and y are related by the equation $y = px + \frac{r}{px}$,

where p and r are constants.

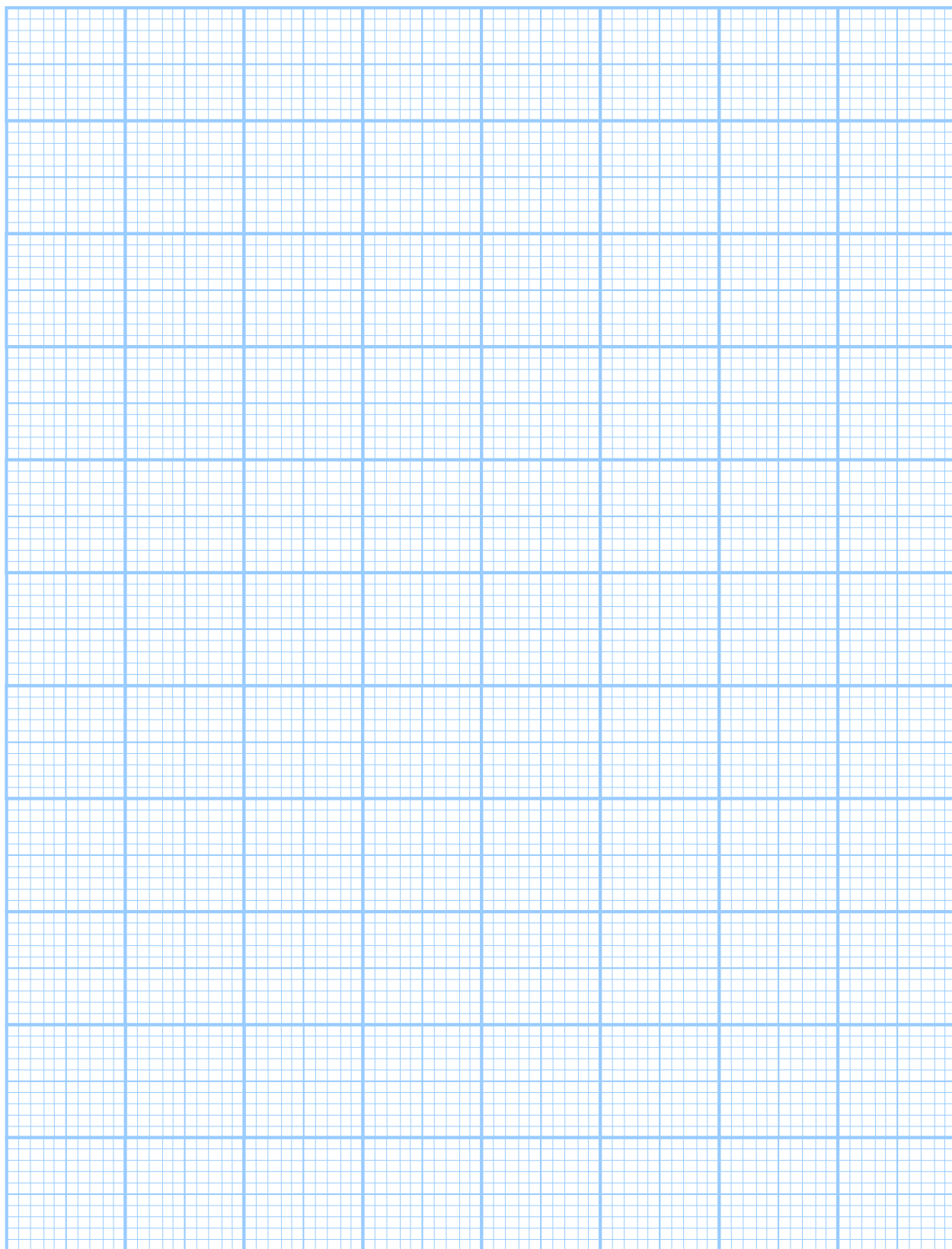
x	1.0	2.0	3.0	4.0	5.0	5.5
y	5.5	4.7	5.0	6.5	7.7	8.4

- (b) Plot xy against x^2 by using a scale of 2 cm to 5 units on both axes.
Hence, draw the line of best fit

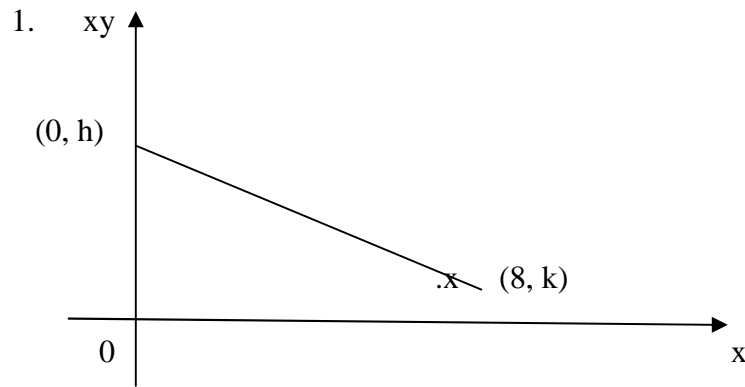
- (b) Use the graph in (a) to find the value of

(i) p

(ii) r



5.0 ASSESSMENT TEST



The above figure shows part of a straight-line graph drawn to represent the equation

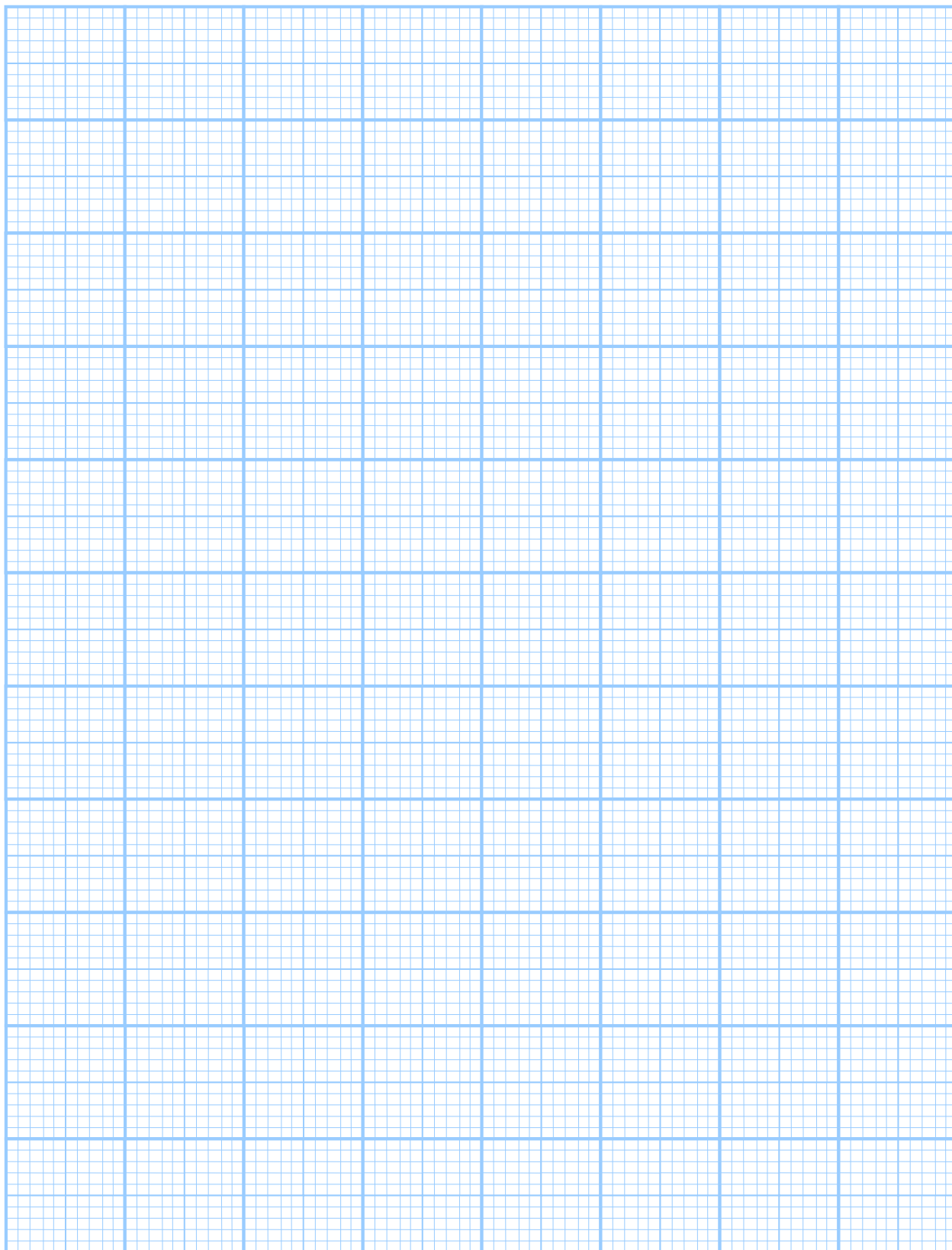
$y = \frac{6}{x} - \frac{1}{2}$ Find the value of h and of k

3. The table below shows some experimental data of two related variable x and y

x	1	2	3	4	5
Y	- 1.5	9	43.5	114	232.5

It is know that x and y are related by an equation in the form
 $y = ax^3 - bx$, where a and b are constants.

- a) Change the equation to the linear form and hence draw the straight line graph for values of x and y
- b) From your graph, I).determine the values of a and b
II).find the value of y when x = 3.5



6.0 ANSWERS

Exercises 2

- a) $y = -x + 9$
- b) $y = 3x - 3$
- c) $y = -x + 7$
- d) $y = 2x - 8$

Exercises 4

	Y	X	m	c
a)	$.y^2$	x	a	b
b)	$\frac{y}{x}$	x	a	b
c)	$\frac{1}{y}$	$\frac{1}{x}$	$\frac{b}{a}$	$\frac{1}{a}$
d)	$\frac{y^2}{x}$	x	5	3
e)	$y\sqrt{x}$	x	3	5
f)	$\lg y$	x	$\lg b$	$\lg a$
g)	\sqrt{y}	x	$\frac{2}{a}$	$\frac{2b}{a}$

SPM Questions

- 1) $p = -2$, $q = 13$
- 2) $h = 3$, $k = 4$
- 3) a) $\lg y = 4\lg x + \lg k$
b) $k = 100$, $h = 11$
- 4) $p = 1.259$, $k = 1.109$
- 5) $p = 1.82$, $k = 1.307$
- 6) $P = 1.37$, $r = 5.48$

Exercises 3

- a) i) $k = 5$ b) i) $k = 3$
ii) $k = 1$ ii) $k = 3$
iii) $k = 5$ iii) $k = 6$

Exercises 5

- a) $a = 1$, $b = 2$
- b) $a = 11$, $b = 8$
- c) $a = 3$, $b = 3$

Exercises 5

- 1) $a = -1$, $b = 10$
- 2) $a = 0.5$, $b = 2.8$

Assessment test

- 1) $h = 6$, $k = 2$
- 2) a) $\frac{y}{x} = ax^2 - b$
b) i. $a = 2$, $b = 3.6$
ii. $y = 73.5$