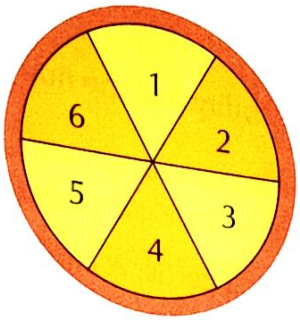
Exercise 15A

BASIC LEVEL

1. A dart board is divided into 6 equal sectors. When a dart lands on it, the number of the sector on which it lands is noted. Write down the sample space and state the total number of possible outcomes.



2. For each of the following experiments, write down the sample space and state the total number of possible outcomes.
   (a) Tossing a fair tetrahedral die with faces labelled 2, 3, 4 and 5 respectively
   (b) Drawing a card at random from a box containing ten identical cards labelled A, B, C, D, E, F, G, H, I, J
   (c) Drawing a disc at random from a bag containing 5 identical red discs, 3 identical blue discs and 2 identical green discs
   (d) Picking a letter at random from a box containing identical cards with letters that spell the word 'TEACHER'
   (e) Choosing a three-digit number at random

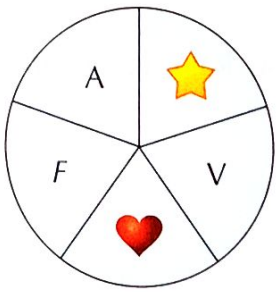
3. An 8-sided fair die with faces labelled 2, 3, 3, 4, 7, 7, 7 and 9 is rolled once. Find the probability of getting
   (i) a '7',
   (ii) a '3' or a '4',
   (iii) a number less than 10,
   (iv) a number which is not '2'.

4. A card is drawn at random from a box containing some cards numbered 10, 11, 12, ..., 22. Find the probability of drawing
   (i) an even number,
   (ii) a number between 13 and 19 inclusive,
   (iii) a prime number that is less than 18,
   (iv) a number greater than 22,
   (v) a number that is divisible by 4.

5. A card is drawn at random from a standard pack of 52 playing cards. Find the probability of drawing
   (i) the ace of spades,
   (ii) a heart or a club,
   (iii) a picture card,
   (iv) a non-picture card.

6. Each of the letters of the word 'PROBABILITY' is written on a card. All the cards are well-shuffled and placed face down on a table. A card is turned over. Find the probability that the card shows
   (i) the letter 'A',
   (ii) the letter 'B',
   (iii) a vowel,
   (iv) a consonant.

7. A spinner is divided into 5 equal sectors. When the spinner is spun, what is the probability that the pointer will stop at a sector whose label is
   (i) ♥?
   (ii) a letter of the English alphabet?
   (iii) a vowel?
   (iv) a consonant?



8. A bag contains 4 pieces of candy – caramel, chocolate, gummies and licorice. A piece of candy is removed at random from the bag. Find the probability that the candy is
   (i) a caramel,
   (ii) either a chocolate or a gummy,
   (iii) not a licorice.

9. An envelope contains 40 shopping vouchers, of which 25 vouchers each have a value of $50 and 15 vouchers each have a value of $100. Amirah picks a voucher at random from the envelope. Find the probability that the voucher has a value of $100.

10. A group of 30 people consisting of 9 men, 6 women, 12 boys and 3 girls, are waiting to get their passport photographs taken. A person is selected at random from the group. Find the probability that the person is

(i) a male,

(ii) either a woman, a boy or a girl.

INTERMEDIATE LEVEL

11. A two-digit number is chosen at random. Find the probability that the number is

(i) less than 20,

(ii) a perfect square.

12. Two Joker cards are added to a standard pack of 52 playing cards. A card is then drawn at random from the 54 cards. Find the probability of drawing

(i) a red card,

(ii) a two,

(iii) a joker,

(iv) a queen or a king.

Note: A Joker card is neither a black nor a red card.

13. All the clubs are removed from a standard pack of 52 playing cards. A card is drawn at random from the remaining cards. Find the probability of drawing

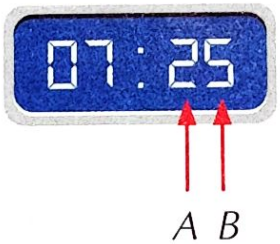
(i) a black card,

(ii) a diamond,

(iii) a picture card,

(iv) a card which is not an ace.

14. Raj wakes up in the morning and notices that his digital clock reads 07 25.



After noon, he looks at the clock again. What is the probability that

(i) the number in column *A* is a 4?

(ii) the number in column *B* is an 8?

(iii) the number in column *A* is less than 6?

(iv) the number in column *B* is greater than 5?

15. A box contains 2 dozen pairs of contact lenses, of which 8 pairs are tinted. A pair of contact lenses is drawn at random from the box. Find the probability that it is not tinted.

16. The table shows the number of each type of school personnel at a school.

Personnel	Management	Teaching	Laboratory	Administrative	Maintenance
Number of personnel	26	62	8	9	12

(a) If a school personnel is selected at random, find the probability that the school personnel is
   (i) a teacher,
   (ii) a management staff,
   (iii) an administrative or a maintenance staff.

(b) Two teachers and an administrative staff resign from the school. A school personnel is selected at random from the remaining staff. Find the probability that the school personnel is
   (i) an administrative staff,
   (ii) not a laboratory staff.

17. There are a total of 117 pairs of socks in a clothes bin. Each pair of socks is placed in a bag. The probabilities of selecting a yellow pair of socks and a grey pair of socks at random from the bin are $\frac{2}{9}$ and $\frac{3}{13}$ respectively. Find the number of pairs of socks in the bin which are
   (i) yellow,
   (ii) neither yellow nor grey.

ADVANCED LEVEL

18. An IQ test consists of 80 multiple-choice questions. A question is selected at random. Find the probability that the question number
   (i) contains only a single digit,
   (ii) is greater than 67,
   (iii) contains exactly one '7',
   (iv) is divisible by both 2 and 5.

19. Each of the numbers 2, 3, 5 and 7 is written on a card. Two of the cards are drawn at random to form a two-digit number. Find the probability that the two-digit number is
   (i) divisible by 4,
   (ii) a prime number.

20. A biased tetrahedral die with faces labelled 1, 2, 3 and 4 is rolled once. The chance of getting a '3' is twice that of getting a '1'. The chance of getting a '2' is thrice that of getting a '3'. There is an equal chance of getting a '2' and a '4'. Find the probability of getting a prime number.

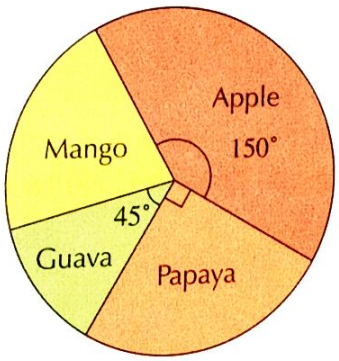
Exercise 15B

BASIC LEVEL

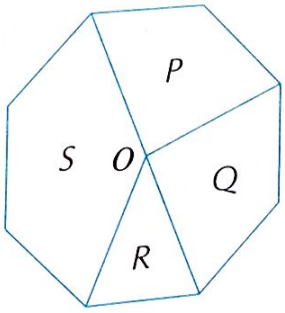
1. A class of 30 students consists of 8 Chinese girls, 3 Malay girls, 1 Indian girl, 11 Chinese boys, 4 Malay boys and 3 Indian boys. If a student is chosen at random to take part in a survey, find the probability that the student is
   (i) a girl,
   (ii) not a Chinese,
   (iii) not an Indian boy,
   (iv) a Eurasian.

2. Shirley has 5 novels and 5 comic books in her bag. Three of her books are in Japanese, 2 of which are comic books. The rest of her books are in English. If a book is chosen at random from her bag, find the probability of choosing
   (i) a book in Japanese,
   (ii) a novel which is in English.

3. A survey is conducted to find out which of the four fruits, apple, papaya, guava and mango, the students in a class prefer. The pie chart shows the results of the survey. A student is selected at random. Find the probability that the student prefers
   (i) apple,
   (ii) mango,
   (iii) papaya or guava.



4. A regular octagon is divided into 4 regions, where O is its centre. A point is selected at random in the octagon. Find the probability that the point lies in
   (i) region R,
   (ii) region S,
   (iii) region P or Q.



5. There are 15 girls and x boys at a school parade square.
   (i) Write down an expression, in terms of x, for the total number of students at the school parade square.
   (ii) A student is selected at random. Write down an expression, in terms of x, for the probability that the student is a girl.
   (iii) Given that the probability in **(ii)** is $\frac{1}{5}$, find the value of x.

INTERMEDIATE LEVEL

6. A class of 38 students went on a short trip to Bangkok. Of the 18 boys, 6 of them checked in their luggage at the airport. 8 of the girls did not check in their luggage. If a student is chosen at random, find the probability that the student
   (i) is a girl who did not check in her luggage,
   (ii) checked in his/her luggage.

7. **(a)** A class has 16 boys and 24 girls. Of the 16 boys, 3 are left-handed. Of the 24 girls, 2 are left-handed. If a student is chosen at random to clean the whiteboard, find the probability that the student is
   (i) a boy,
   (ii) left-handed.
   (b) The student chosen to clean the whiteboard in **(a)** is a girl who is not left-handed. Another student is selected at random from the remaining students to borrow the visualiser from the class next door. Find the probability that the student is
   (i) a boy who is left-handed,
   (ii) a girl who is not left-handed.

8. Santa Claus has $(3h + 11)$ red presents and $(h + 5)$ white presents in his stocking. Ethan selects a present at random from the stocking. Given that the probability that he obtains a red present is $\frac{19}{26}$, find the value of h.

9. Some patients participated in a clinical trial for a new drug to treat osteoporosis. A patient is selected at random. The probability that the patient had no change in his bone mass density is $\frac{7}{13}$, the probability that he had a slight reduction in his bone mass density is $\frac{1}{k}$ and the probability that he had a significant reduction in his bone mass density is $\frac{1}{2k}$. Find the value of k.

10. A carton contains 15 toothbrushes, of which p have soft bristles. After 5 more toothbrushes with soft bristles are added to the carton, the probability of drawing a toothbrush with soft bristles becomes $\frac{3}{4}$. Find the value of p.

11. There are 23 boys and 35 girls on the school's track and field team. After q boys and $(q+4)$ girls graduate at the end of this year, the probability of selecting a boy at random to represent the school for an event becomes $\frac{2}{5}$. Find the value of q.

ADVANCED LEVEL

12. A bag contains 40 balls, some of which are red, some of which are yellow and the rest are black. The probabilities of drawing a red ball and a yellow ball at random from the bag are $\frac{1}{4}$ and $\frac{2}{5}$ respectively.

(i) Find the probability of drawing a black ball at random from the bag.

$(2x+1)$ red balls and $(x+2)$ yellow balls are added to the bag while $(x-3)$ black balls are removed from the bag. The probability of drawing a yellow ball at random from the bag is now $\frac{3}{7}$. Find

(ii) an expression, in terms of x, for the total number of balls in the bag now,

(iii) the number of yellow balls in the bag now.

13. There are 50 students in an auditorium, of which $2x$ are boys and y are girls. After $(y-6)$ boys leave the auditorium and $(2x-5)$ girls enter the auditorium, the probability of selecting a girl at random becomes $\frac{9}{13}$. Find the value of x and of y.

1. **Probability** is a measure of chance.

2. A **sample space** is the collection of all the possible outcomes of a probability experiment.

3. In a probability experiment with m *equally likely* outcomes, if k of these outcomes favour the occurrence of an event E, then the probability, $\mathrm{P}(E)$, of the event happening is given by:

$$\mathrm{P}(E) = \frac{\text{Number of favourable outcomes for event } E}{\text{Total number of possible outcomes}} = \frac{k}{m}$$

4. For any event E, $0 \leqslant \mathrm{P}(E) \leqslant 1$.

   - $\mathrm{P}(E) = 0$ if and only if E is an *impossible* event, i.e. it will *never* occur.
   - $\mathrm{P}(E) = 1$ if and only if E is a *certain* event, i.e. it will *definitely* occur.

5. For any event E, $\mathrm{P}(\text{not } E) = 1 - \mathrm{P}(E)$.