The CitySchool
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Quadratic Equations Explanation

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**Class: 9
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An example of a **Quadratic Equation**:



Quadratic Equations make nice curves, like this one:



Name

The name **Quadratic** comes from "quad" meaning square, because the variable gets [squared](https://www.mathsisfun.com/square-root.html) (like **x2**).

It is also called an "Equation of [Degree](https://www.mathsisfun.com/algebra/degree-expression.html) 2" (because of the "2" on the **x**)

Standard Form

The **Standard Form** of a Quadratic Equation looks like this:



|  |
| --- |
| * **a**, **b** and **c** are known values. **a** can't be 0.
* "**x**" is the [**variable**](https://www.mathsisfun.com/algebra/definitions.html) or unknown (we don't know it yet).
 |

Here are some more examples:

|  |  |  |
| --- | --- | --- |
| **2x2 + 5x + 3 = 0** |   | In this one **a=2**, **b=5** and **c=3** |
|   |   |   |
| **x2 − 3x = 0** |   | This one is a little more tricky:* Where is **a**? Well **a=1**, and we don't usually write "1x2"
* **b = -3**
* And where is **c**? Well **c=0**, so is not shown.
 |
| **5x − 3 = 0** |   | **Oops!** This one is **not**a quadratic equation: it is missing **x2**(in other words **a=0**, which means it can't be quadratic) |

Hidden Quadratic Equations!

So the "Standard Form" of a Quadratic Equation is

ax2 + bx + c = 0

But sometimes a quadratic equation doesn't look like that! For example:

|  |  |  |  |
| --- | --- | --- | --- |
| **In disguise** | **→** | **In Standard Form** | **a, b and c** |
| **x2 = 3x − 1** | Move all terms to left hand side | **x2 − 3x + 1 = 0** | a=1, b=−3, c=1 |
| **2(w2 − 2w) = 5** | [Expand](https://www.mathsisfun.com/algebra/expanding.html) (undo the [brackets](https://www.mathsisfun.com/algebra/brackets.html)), and move 5 to left | **2w2 − 4w − 5 = 0** | a=2, b=−4, c=−5 |
| **z(z−1) = 3** | Expand, and move 3 to left | **z2 − z − 3 = 0** | a=1, b=−1, c=−3 |

|  |  |  |
| --- | --- | --- |
| Quadratic Graph |   | Have a Play With ItPlay with the "[Quadratic Equation Explorer](https://www.mathsisfun.com/algebra/quadratic-equation-graph.html)" so you can see:* the graph it makes, and
* the solutions (called "roots").
 |

How To Solve It?

The "**solutions**" to the Quadratic Equation are where it is **equal to zero**.

There are usually 2 solutions (as shown in the graph above).

They are also called "**roots**", or sometimes "**zeros**"

There are 3 ways to find the solutions:

1. We can [Factor the Quadratic](https://www.mathsisfun.com/algebra/factoring-quadratics.html) (find what to multiply to make the Quadratic Equation)

2. We can [Complete the Square](https://www.mathsisfun.com/algebra/completing-square.html), or

3. We can use the special **Quadratic Formula**:



Just plug in the values of a, b and c, and do the calculations.

We will look at this method in more detail now.

About the Quadratic Formula

Plus/Minus

First of all what is that plus/minus thing that looks like **±**?

|  |  |  |  |
| --- | --- | --- | --- |
| https://www.mathsisfun.com/images/style/right-arrow.gif | The ± means there are TWO answers:https://www.mathsisfun.com/algebra/images/quadratic-formula-two.gifHere is why we can get two answers: |   | Quadratic Graph |

But sometimes we don't get two real answers, and the "Discriminant" shows why ...

Discriminant

Do you see **b2 − 4ac** in the formula above? It is called the **Discriminant**, because it can "discriminate" between the possible types of answer:

* when **b2 - 4ac** is positive, we get two [Real](https://www.mathsisfun.com/numbers/real-numbers.html) solutions
* when it is zero we get just ONE real solution (both answers are the same)
* when it is negative we get two [Complex](https://www.mathsisfun.com/numbers/complex-numbers.html) solutions

*Complex solutions?* Let's talk about them after we see how to use the formula.

Using the Quadratic Formula

Just put the values of a, b and c into the Quadratic Formula, and do the calculations.

Example: Solve 5x² + 6x + 1 = 0

|  |  |  |
| --- | --- | --- |
| **Coefficients are:** |   | a = 5, b = 6, c = 1 |
|   |   |   |
| **Quadratic Formula:** |   | x = *−b ± √(b2− 4ac)***2a** |
|   |   |   |
| **Put in a, b and c:** |   | x = *−6 ± √(62− 4×5×1)***2×5** |
|   |   |   |
| **Solve**: |   | x = *−6 ± √(36− 20)***10** |
|   |   | x = *−6 ± √(16)***10** |
|   |   | x = *−6 ± 4***10** |
|   |   | x = −0.2 **or** −1 |

|  |  |
| --- | --- |
| 5x^2+6x+1 | **Answer:** x = −0.2 **or** x = −1 And we see them on this graph. |

|  |  |
| --- | --- |
| Check **-0.2**: | 5×(**−0.2**)² + 6×(**−0.2**) + 1 = 5×(0.04) + 6×(−0.2) + 1 = 0.2 − 1.2 + 1 **= 0** |
| Check **-1**: | 5×(**−1**)² + 6×(**−1**) + 1 = 5×(1) + 6×(−1) + 1 = 5 − 6 + 1 **= 0** |

Remembering The Formula

I don't know of an easy way to remember the Quadratic Formula, but a kind reader suggested singing it to "Pop Goes the Weasel":

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **♫**   | ***"x equals minus b*** |  | **♫**   | *"All around the mulberry bush* |
| ***plus or minus the square root*** |   | *The monkey chased the weasel* |
|   | ***of b-squared minus four a c*** |   |   | *The monkey thought 'twas all in fun* |
|   | ***all over two a"*** |   |   | *Pop! goes the weasel"* |

Try singing it a few times and it will get stuck in your head!

Or you can remember this story:

x = *−b ± √(b2− 4ac)***2a**

*"A negative boy was thinking yes or no about going to a party,
at the party he talked to a square boy but not to the 4 awesome chicks.
It was all over at 2 am.*"

Complex Solutions?

When the Discriminant (the value **b2 − 4ac**) is negative we get [Complex](https://www.mathsisfun.com/numbers/complex-numbers.html) solutions ... what does that mean?

It means our answer will include [Imaginary Numbers](https://www.mathsisfun.com/numbers/imaginary-numbers.html). Wow!

Example: Solve 5x² + 2x + 1 = 0

|  |  |  |
| --- | --- | --- |
| **Coefficients** are**:** |   | a = 5, b = 2, c = 1 |
|   |   |   |
| Note that the **Discriminant** is negative: |   | b2 − 4ac = 22 − 4×5×1 = **-16** |
|   |   |   |
| Use the **Quadratic Formula:** |   | x = *−2 ± √(−16)***10** |
|   |   |   |

Summary

* Quadratic Equation in Standard Form: ax2 + bx + c = 0
* Quadratic Equations can be [factored](https://www.mathsisfun.com/algebra/factoring-quadratics.html)
* Quadratic Formula: x = *−b ± √(b2− 4ac)***2a**
* When the Discriminant (**b2−4ac**) is:
	+ positive, there are 2 real solutions
	+ zero, there is one real solution
	+ negative, there are 2 complex solutions