The City School

North Nazimabad Boys Campus
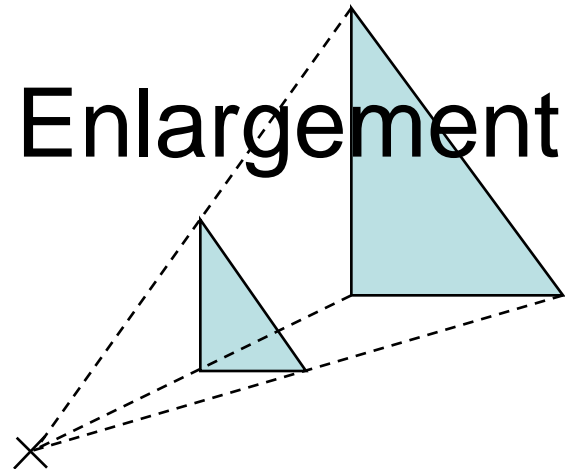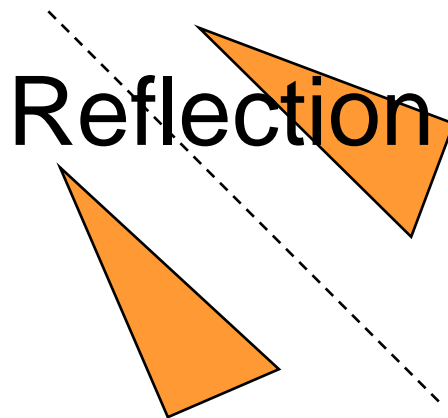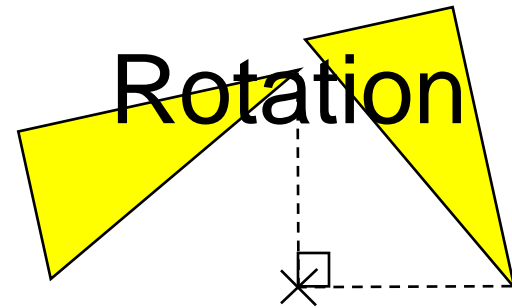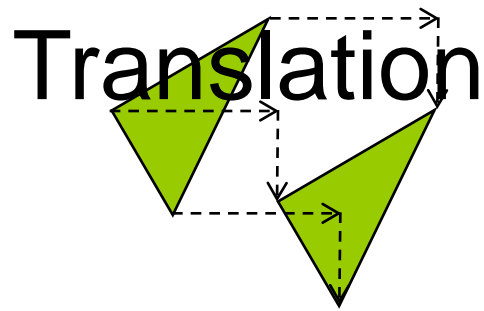
Subject: Maths

Class: 11

Transformations

for GCSE Maths

Translation

Rotation

Reflection
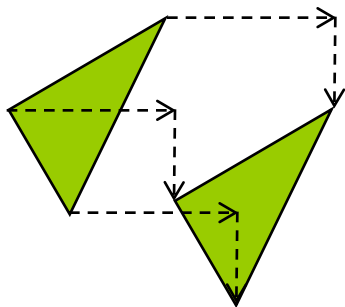
Enlargement

Translation

Translations are usually given in vector form:

$$\begin{pmatrix} 3 \\ -1 \end{pmatrix}$$ means 3 units to the right and 1 unit down.

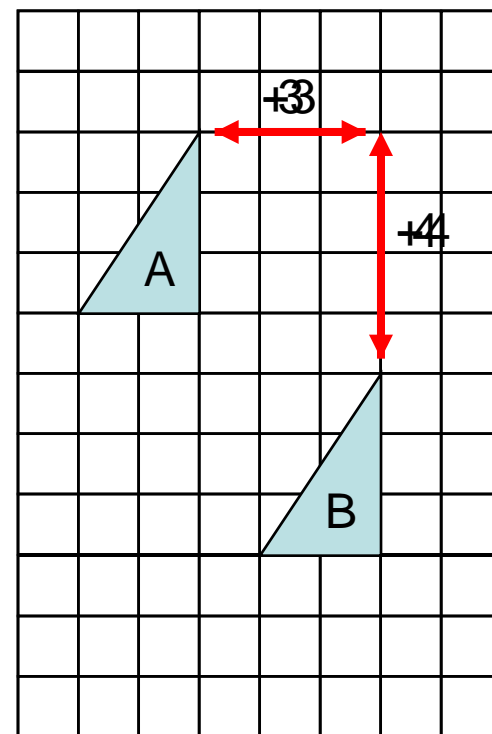So what does $$\begin{pmatrix} -2 \\ 5 \end{pmatrix}$$ mean?

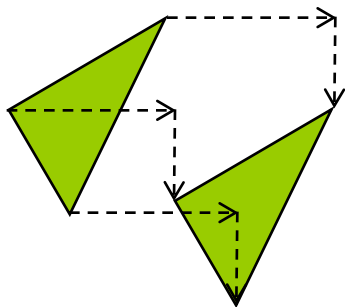What is the vector translating A to B?
What about B to A?

A to B: $$\begin{pmatrix} 3 \\ -4 \end{pmatrix}$$ B to A: $$\begin{pmatrix} -3 \\ 4 \end{pmatrix}$$

After translation, a shape will be
• the same shape and size
• the same way round (orientation)
• in a different position

Translation

- Start by taking one corner of the shape to be translated.

- Draw in the translation vector and mark the position of the corresponding corner on the translated shape.

- Repeat with the other corners until you are able to complete the shape.
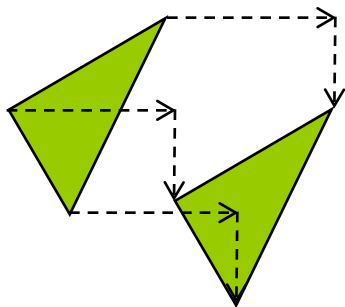
  Now translate shape A with vector $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$

Try the question before you click to continue!

New location of top corner

3 up

2 right

Start here

A'

A
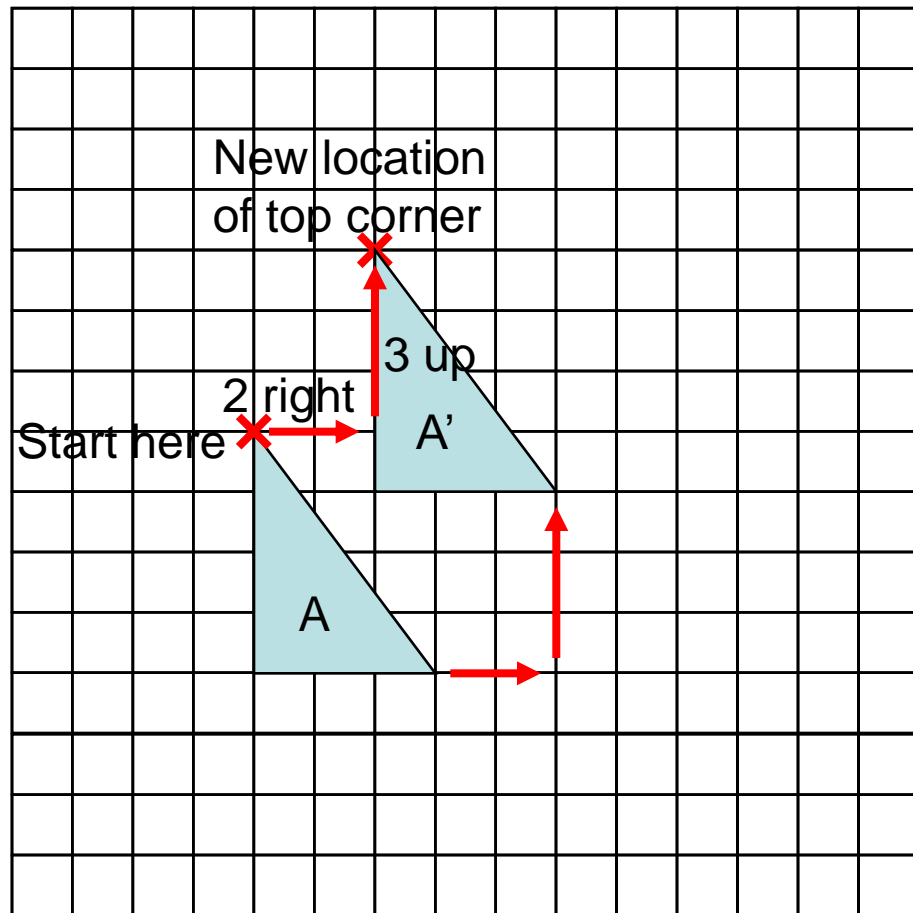
Translation

Now try a couple more!

Translate shape P
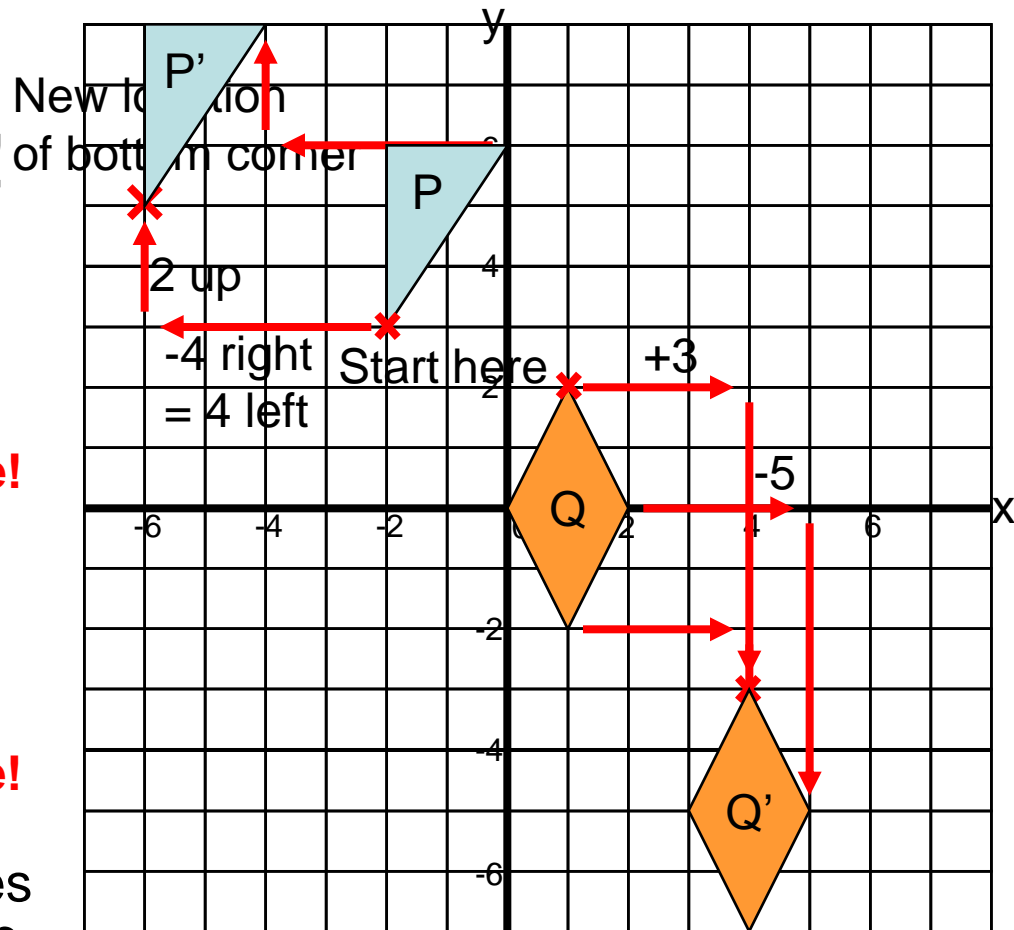with vector $\begin{pmatrix} -4 \\ 2 \end{pmatrix}$

Try it before you click to continue!

Now translate shape Q
with vector $\begin{pmatrix} 3 \\ -5 \end{pmatrix}$

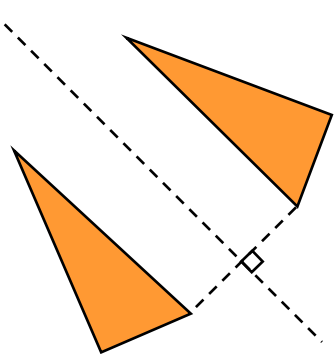**Try it before you click to continue!**

Can you see how the coordinates
of the vertices (corners) change?
Look at the coordinates of one
vertex at a time.

New location
of bottom corner

P'

P

2 up

-4 right
= 4 left

Start here

+3

-5

Q

Q'

y

x

-6   -4   -2   0   2   4   6

2

-2

-4

-6

4

What vector would translate each
shape back to its original position?
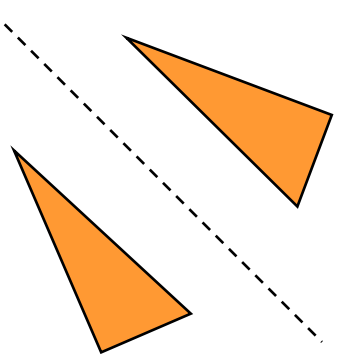
Reflection

- When a shape is reflected, the reflection takes place in a "line of reflection" or "mirror line".

- To draw the reflection, first draw the mirror line.

- Now draw a perpendicular line from one corner of the original shape to the mirror line, and extend it for the same distance on the other side (as on diagram above).

- Repeat with the other corners until you can draw the whole shape.

Reflection

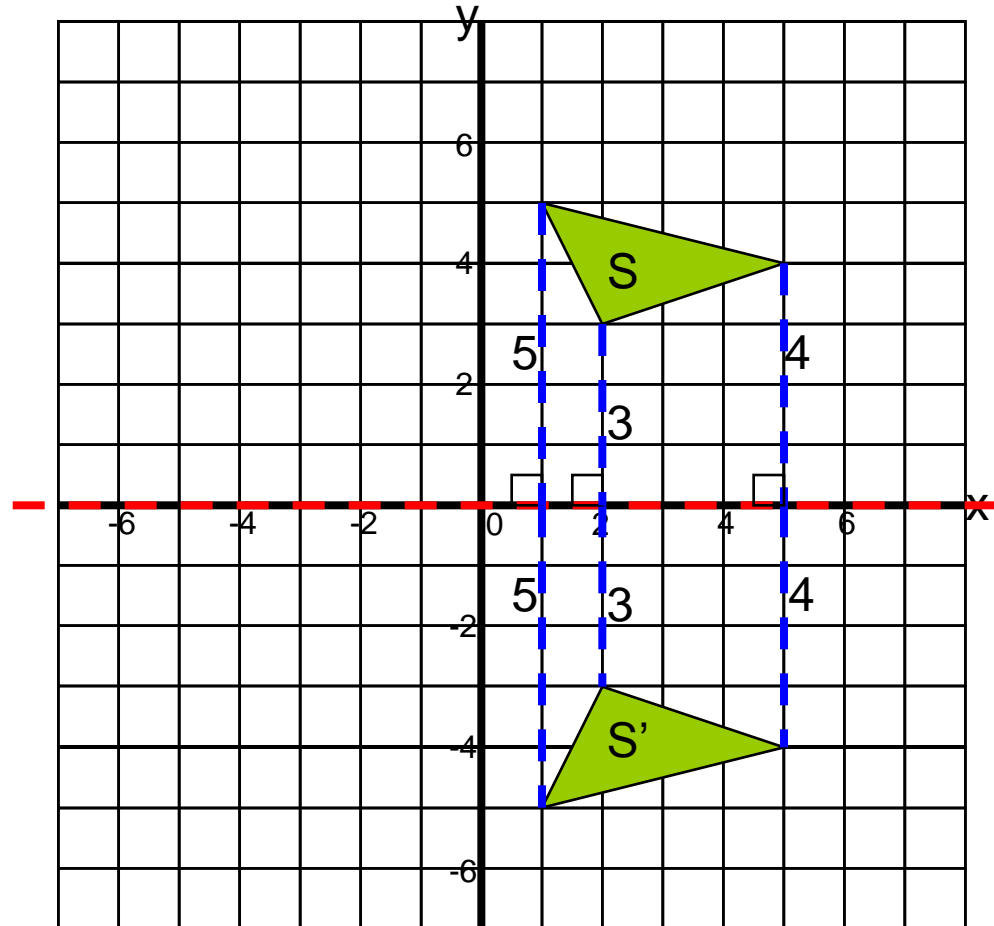Now try reflecting shape S in the line y=0.

Have a go before you click to continue!

Where is the line y=0? Hint: the points (0,0), (1,0), (2,0) etc. are all on it…

…yes, it's the x-axis!

Now draw in your perpendicular lines…
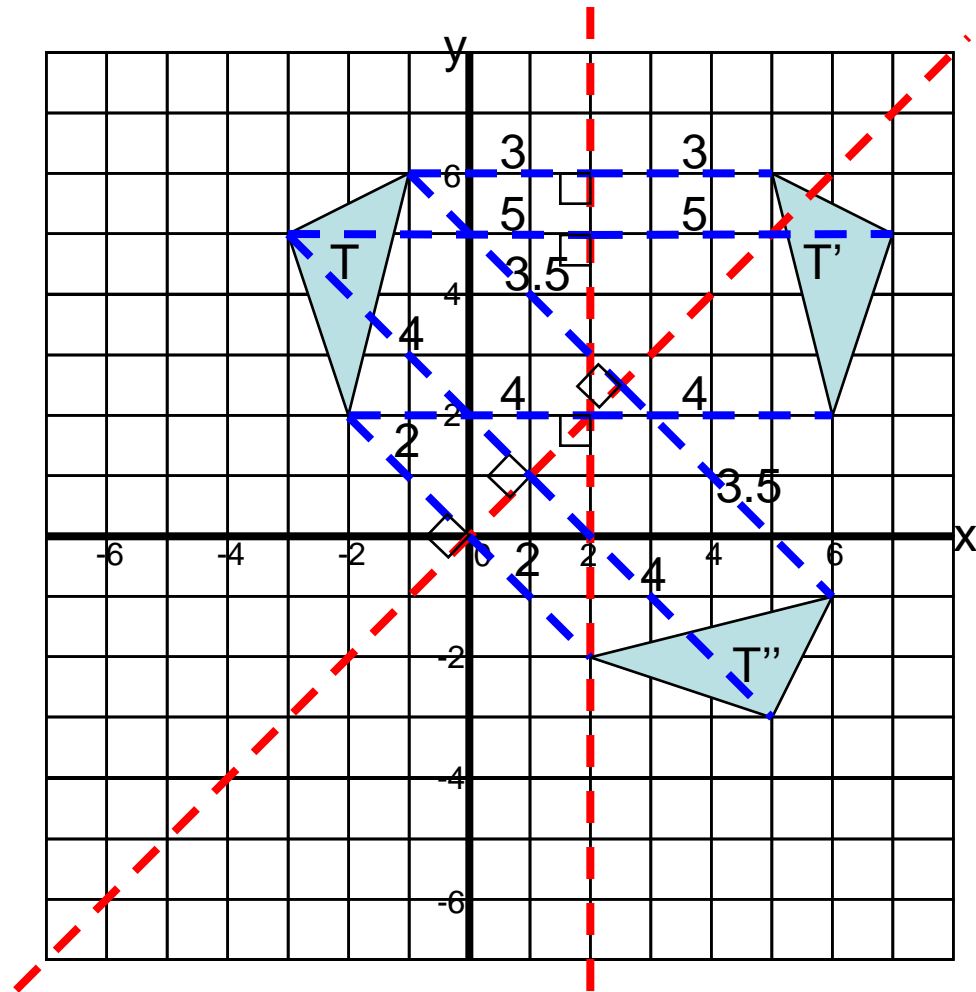
Finally, draw in your transformed shape…

Reflection

- Now reflect shape T in the line x=2…

Try it before clicking to continue!

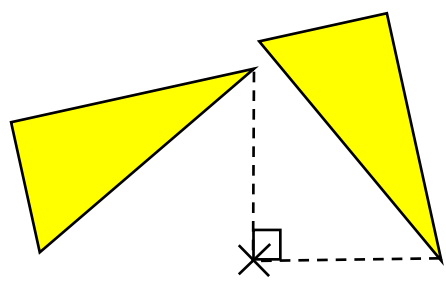- … and then reflect shape T in the line y=x

Try it before clicking to continue!

Rotation

- You need to be able to rotate a shape about a given point
  - clockwise or anticlockwise
  - through 90, 180 or 270 degrees (quarter-, half- or ¾-turn)
- A piece of tracing paper can make this easier!
  - Mark the centre of rotation and draw a straight line connecting it to one corner on the shape
  - Put tracing paper over both the shape AND the centre of rotation
  - Trace the shape and line onto the paper
  - Put your pencil point on the centre of rotation and turn the paper through the appropriate angle
  - Draw the transformed shape in its new position
- Let's give it a try…

Rotation
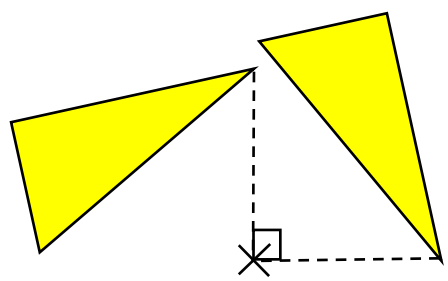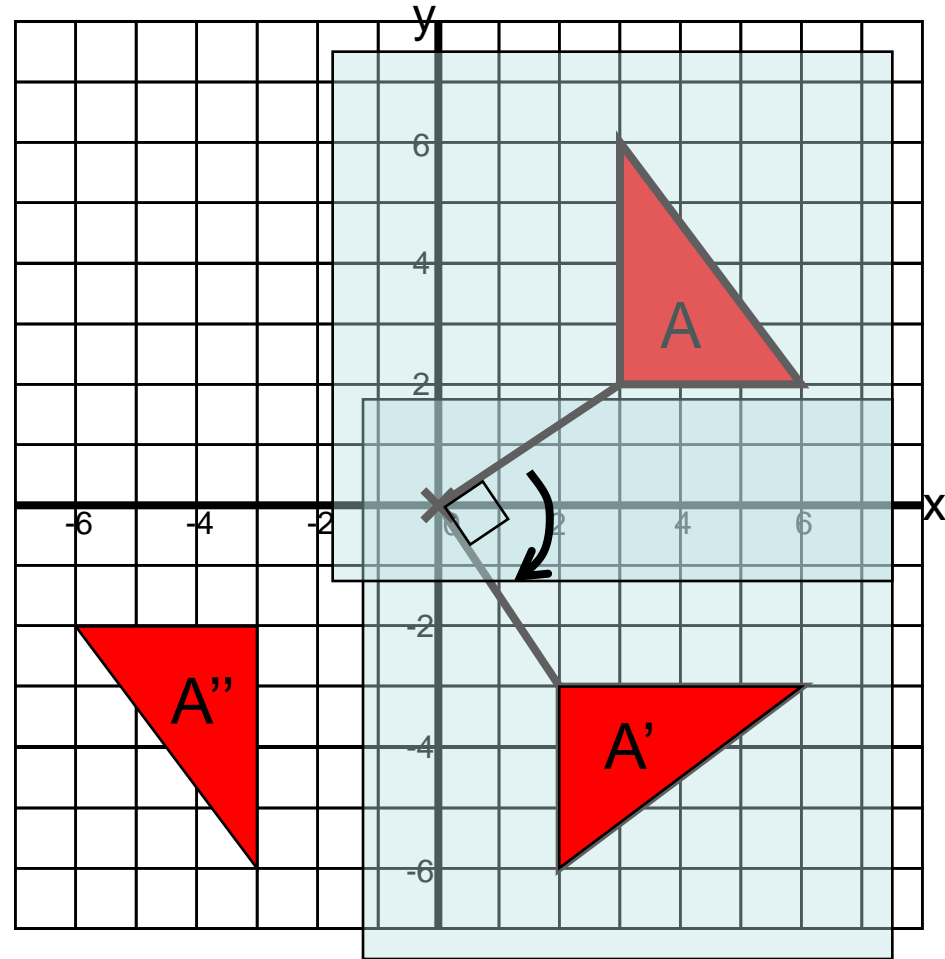
Rotate shape A clockwise through 90° about the origin.

First, mark your centre of rotation at (0,0) and join it to the shape.

Place the tracing paper over the shape and the CoR, and trace.

With your pencil point on the CoR, rotate the tracing paper clockwise through 90°, then transfer the shape to its new position on the grid.
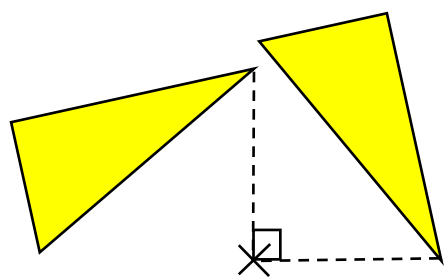
Now try rotating the same shape anticlockwise through 180° about the origin.

Try it, then click to see the answer.

Rotation

Now rotate shape D anticlockwise through 90° about the point (2,-1).
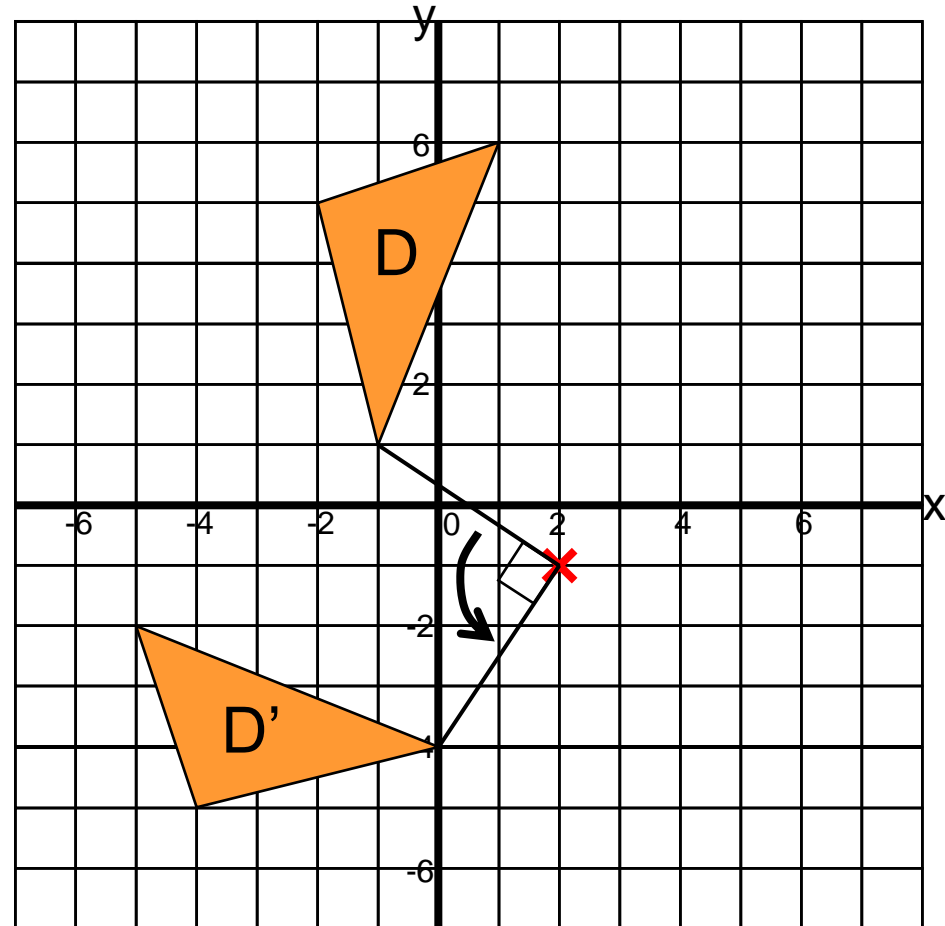
Try it before clicking to continue!

Can you think of another way to describe this transformation?
Click again for the answer…

Rotation <u>clockwise</u> through <u>270°</u> about the point <u>(2, -1)</u>
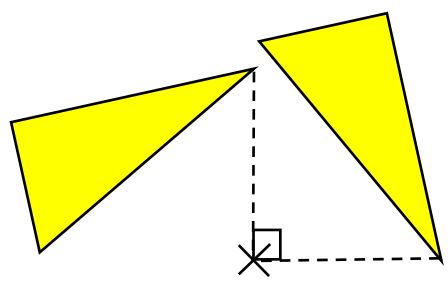
What transformation would return the shape to its original position?

Rotation <u>clockwise</u> through <u>90°</u> (or <u>anticlockwise</u> through <u>270°</u>) about the point <u>(2, -1)</u>
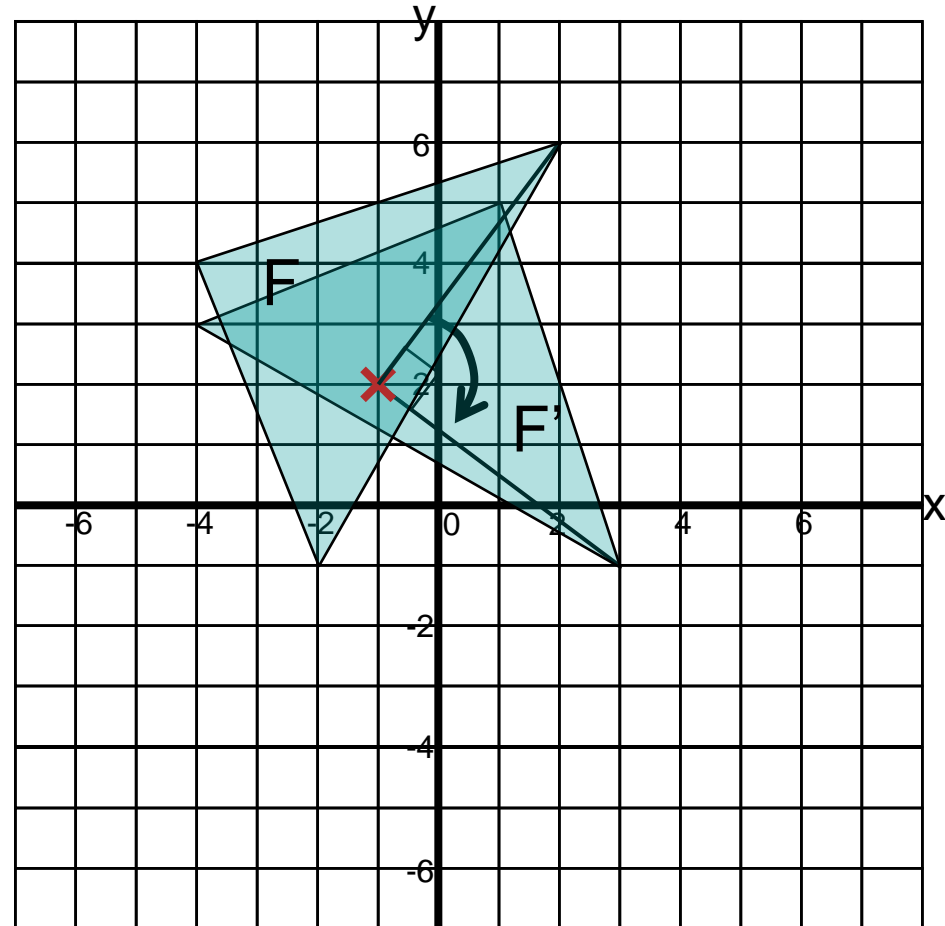
Rotation

Sometimes the centre of rotation can be *inside* the shape.
Try rotating shape F clockwise through 90° about the point (-1,2).

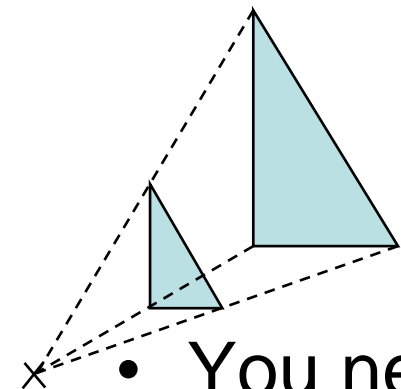**Have a go before clicking to see the answer!**
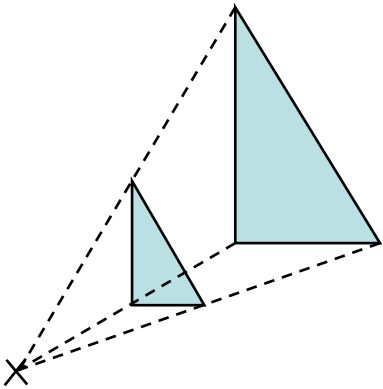
Enlargement

- You need to be able to enlarge a shape about a given point (the centre of enlargement) with a given scale factor

- If the scale factor is more than 1, the shape will get bigger
  - e.g. a scale factor of 3 will result in a shape with sides 3 times as long (and 9 times the area!)

- If the scale factor is less than 1, the shape will get smaller – but it's still called an enlargement!
  - e.g. a scale factor of ½ will result in a shape with sides only half as long (and ¼ of the area!)
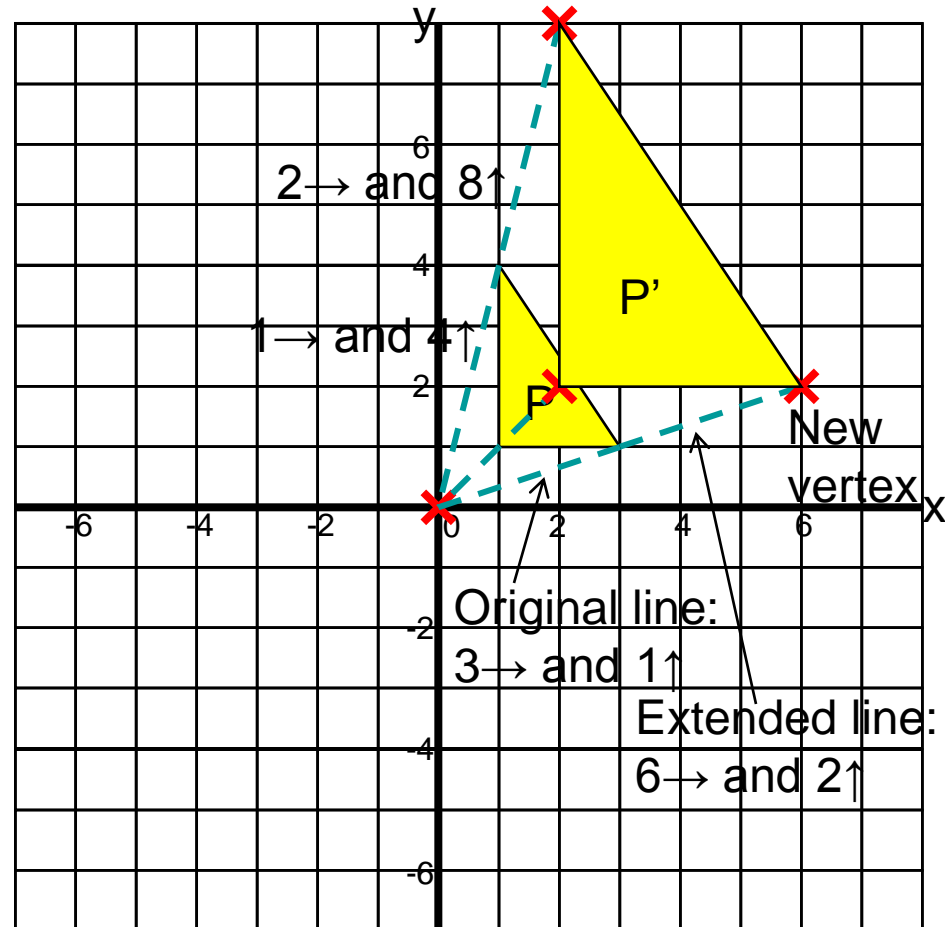
Enlargement

Enlarge shape P about the origin with scale factor 2.

First, mark the centre of enlargement.

Now draw a line from the CoE to one vertex (corner) on the shape.

Multiply the length of line by the S.F. and extend it to find the position of the corresponding vertex on the enlarged shape.
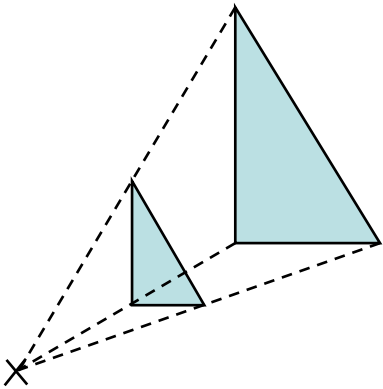
Repeat with the other vertices until you have enough information to draw the new shape.

2→ and 8↑

P'

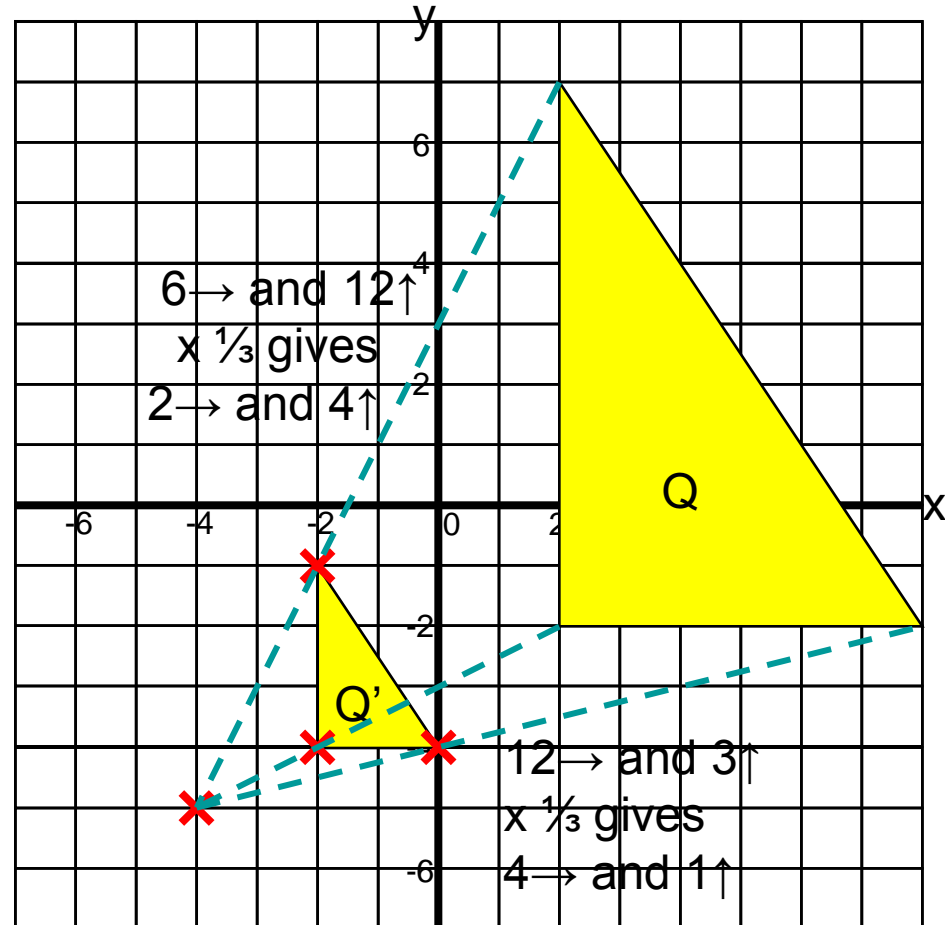1→ and 4↑

P

New vertex

Original line:
3→ and 1↑

Extended line:
6→ and 2↑

Enlargement

Now enlarge shape Q about the point (-4, -5) with scale factor ⅓.

Try it before clicking to see the answer!

6→ and 12↑
x ⅓ gives
2→ and 4↑

Q

Q'

12→ and 3↑
x ⅓ gives
4→ and 1↑
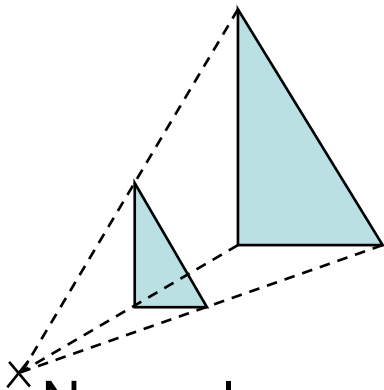
Enlargement

Now enlarge shape R about the point (-2, 1) with scale factor 3.

Final question... **Try it before clicking to continue!**
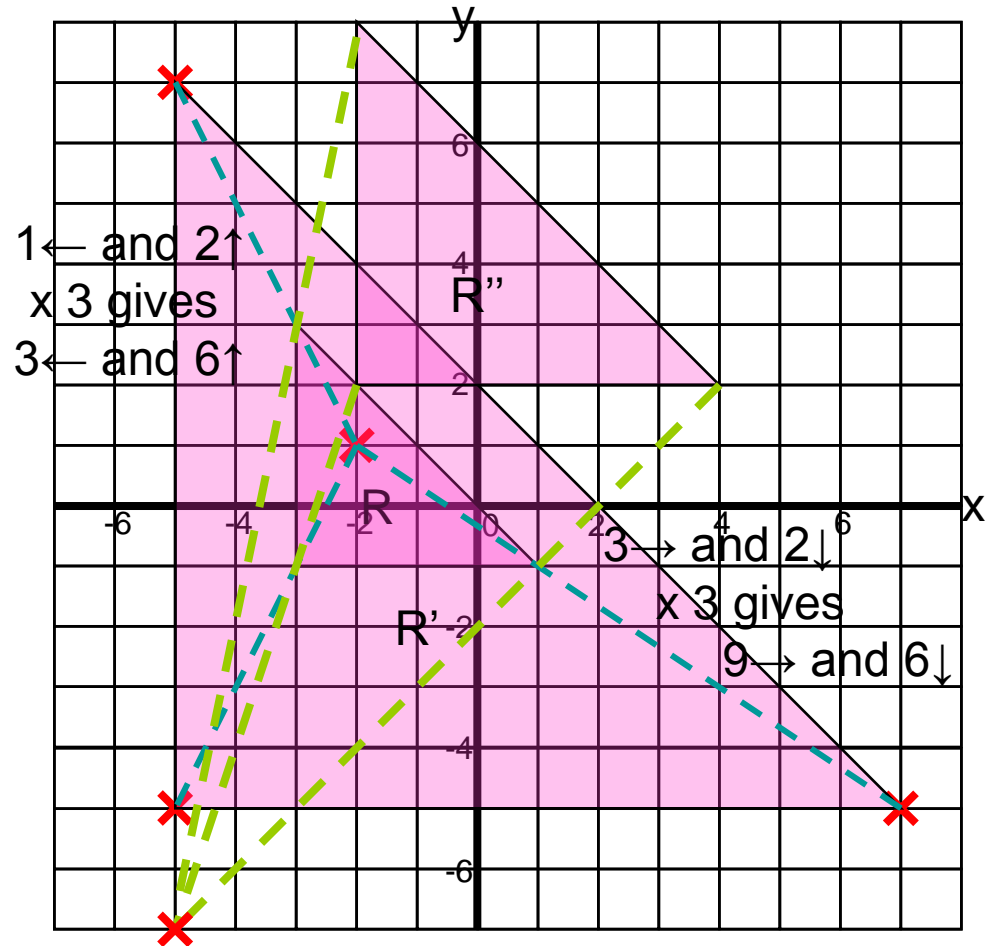
R" is also an enlargement of shape R. Can you work out the scale factor and find the centre of enlargement?

Compare the side lengths:
Scale factor = new length
old length

So scale factor = 6/4 = 1.5

Now draw lines through the corresponding vertices of the two shapes, and extend them back until they meet; this point is the CoE.

1← and 2↑
x 3 gives
3← and 6↑

R"

R

3→ and 2↓
x 3 gives
9→ and 6↓

R'

Centre of enlargement = (-6,-7)