Exam-style questions

6

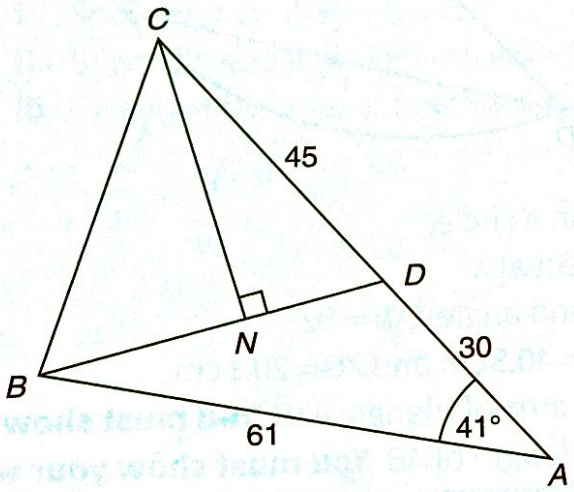


Diagram I

In Diagram I, the point D lies on AC and N is the foot of the perpendicular from C to BD.

$AB = 61$ m, $AD = 30$ m and $DC = 45$ m.

Angle $BAC = 41°$.

a Calculate BD.

b Show that, correct to the nearest square metre, the area of triangle BDA is 600 m^2.

c Explain why $\frac{\text{area of } \Delta BCD}{\text{area of } \Delta BDA} = \frac{3}{2}$.

d Calculate the area of triangle BCD.

e Hence calculate CN.

f

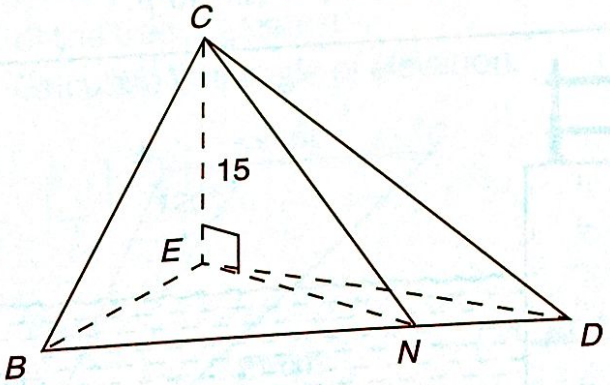


Diagram II

The same points B, C, D and N lie on a sloping plane.

The point E is 15 m vertically below C.

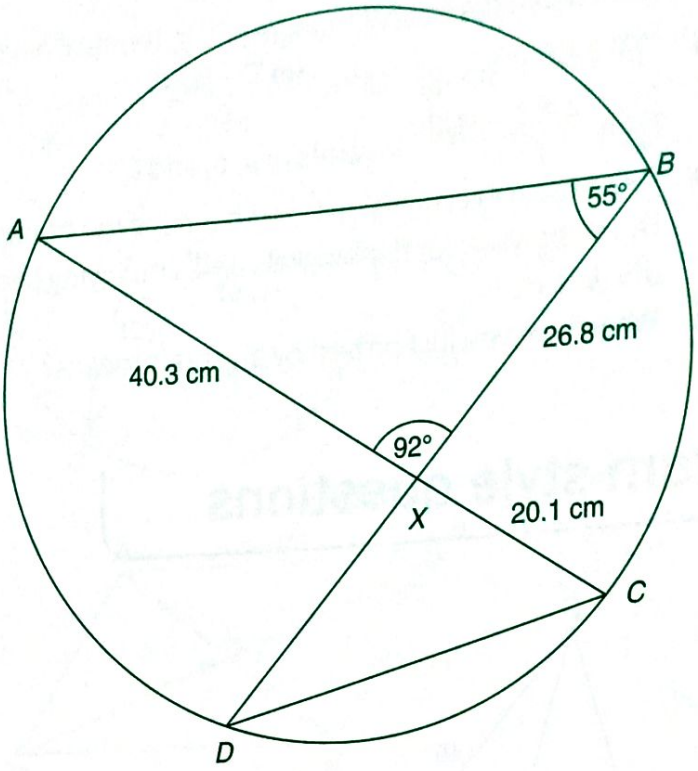
The points B, E, D and N lie on a horizontal plane.

Diagram II represents this information.

Calculate the angle of elevation of C from N.

(4024 paper 02 Q9 June 2007)

7



NOT TO SCALE

A, B, C and D lie on a circle.
AC and BD intersect at X.
Angle $ABX = 55°$ and angle $AXB = 92°$.
$BX = 26.8$ cm, $AX = 40.3$ cm and $XC = 20.1$ cm.

a Calculate the area of triangle AXB. **You must show your working.**

b Calculate the length of AB. **You must show your working.**

c Write down the size of angle ACD. Give a reason for your answer.

d Find the size of angle BDC.

e Write down the geometrical word which completes the statement
'Triangle AXB is _____ to triangle DXC.'

f Calculate the length of XD. **You must show your working.** (0580 paper 04 Q3a June 2007)

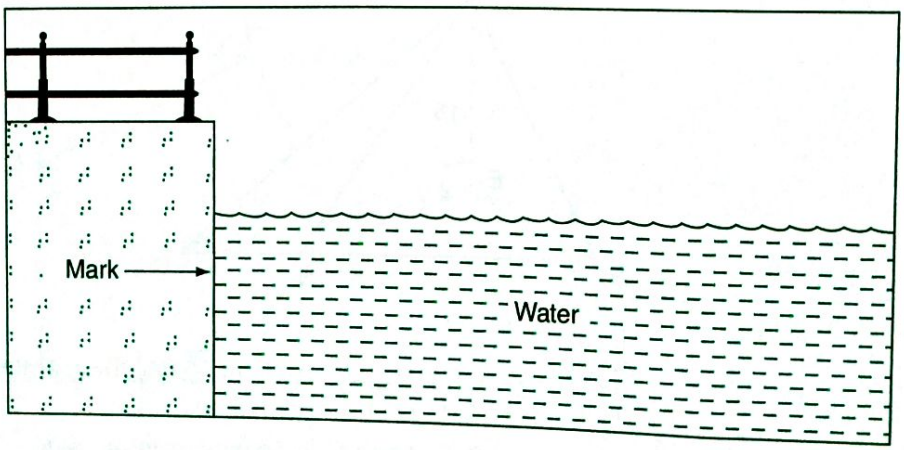
8 **a** Use your calculator to work out

$$\frac{1-(\tan 40°)^2}{2(\tan 40°)}.$$

b Write your answer to part (a) in standard form. (0580 paper 02 Q2 June 2007)

9 Calculate the value of $(\cos 40°)^2 + (\sin 40°)^2$. (0580 paper 02 Q4 June 2005)

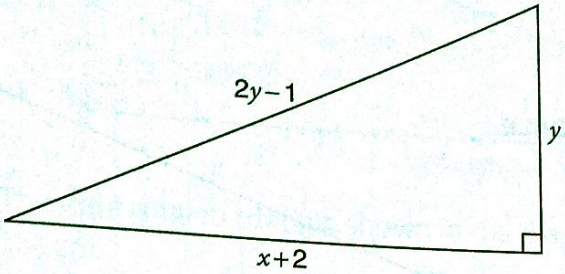
10



The height, h metres, of the water, above a mark on a harbour wall, changes with the tide.
It is given by the equation
$h = 3\sin(30t)°$
where t is the time in hours after midday.

a Calculate the value of h at midday.
b Calculate the value of h at 1900.
c Explain the meaning of the negative sign in your answer. (0580 paper 02 Q17 June 2005)

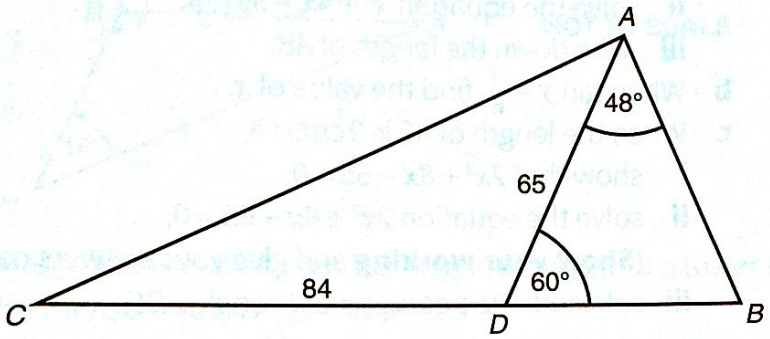
11



NOT TO SCALE

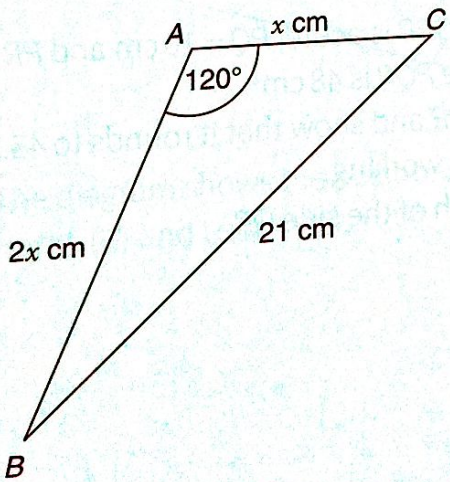
The diagram shows a right-angled triangle.
The lengths of the sides are given in terms of y.
i Show that $2y^2 - 8y - 3 = 0$.
ii Solve the equation $2y^2 - 8y - 3 = 0$, giving your answers to 2 decimal places.
iii Calculate the area of the triangle. (0580 paper 04 Q8b June 2006)

12



A, B, C and D are four points on horizontal ground.
CBD is a straight line.
$AD = 65$ m and $CD = 84$ m.
$D\hat{A}B = 48°$ and $A\hat{D}B = 60°$.
a Calculate AB.
b Calculate the area of triangle ACD.
c Calculate AC.
d A vertical tree of height 35 m stands at A.
P is the point on the line BC such that the angle of elevation from the line BC to the top of the tree is greatest.
Calculate this angle of elevation. (4024 paper 21 Q9 November 2012)

13



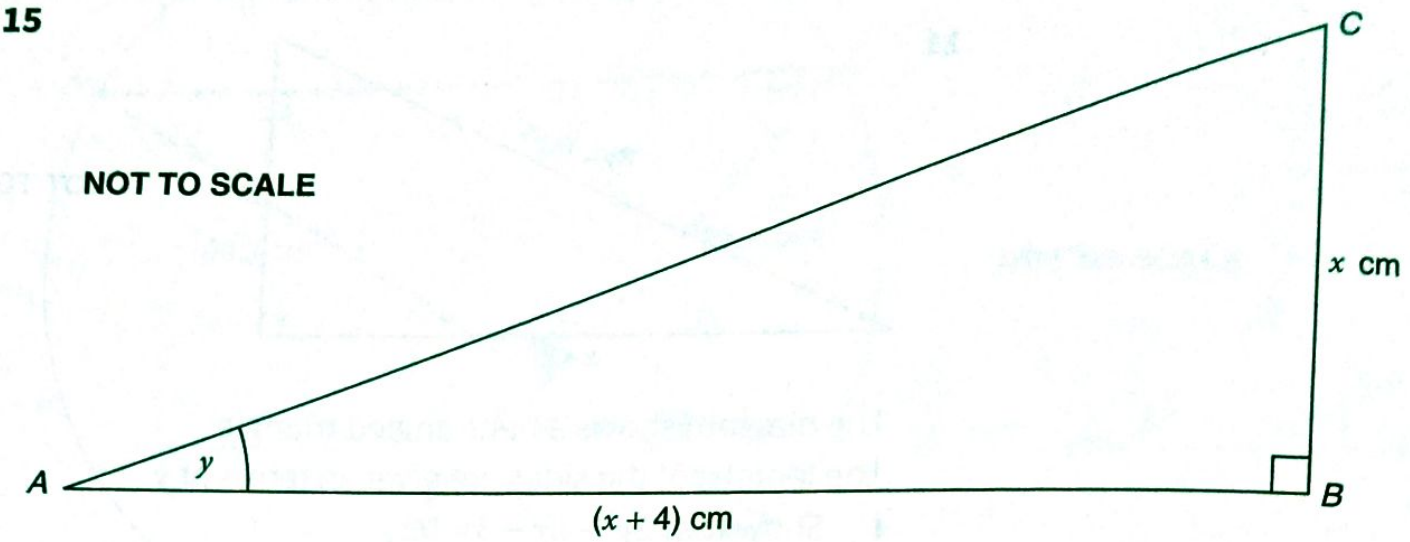
NOT TO SCALE

In triangle ABC, $AB = 2x$ cm, $AC = x$ cm, $BC = 21$ cm and angle $BAC = 120°$.
Calculate the value of x. (0580 paper 21 Q11 June 2008)

14 $\sin x° = 0.866\,03$ and $0 \leqslant x \leqslant 180$.
Find the two values of x.

(0580 paper 21 Q6 November 2008)

15



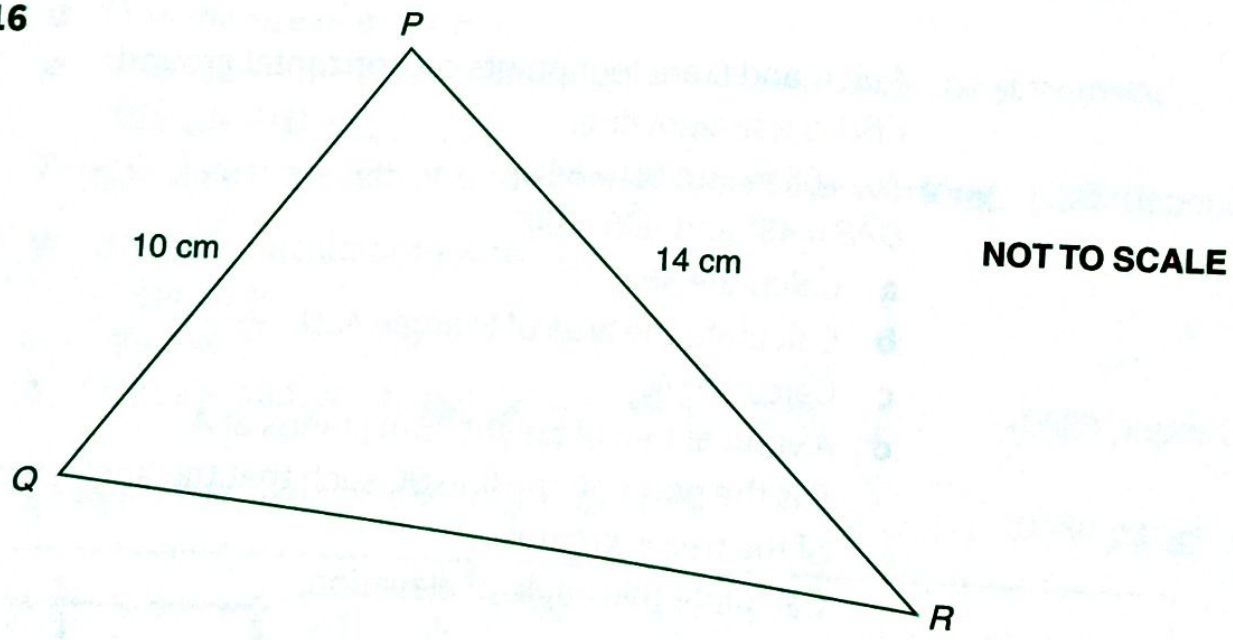
a When the area of triangle ABC is 48 cm^2,
- **i** show that $x^2 + 4x - 96 = 0$,
- **ii** solve the equation $x^2 + 4x - 96 = 0$,
- **iii** write down the length of AB.

b When $\tan y = \frac{1}{6}$, find the value of x.

c When the length of AC is 9 cm,
- **i** show that $2x^2 + 8x - 65 = 0$,
- **ii** solve the equation $2x^2 + 8x - 65 = 0$,
(**Show your working** and give your answers correct to 2 decimal places.)
- **iii** calculate the perimeter of triangle ABC.

(0580 paper 04 Q2 November 2008)

16



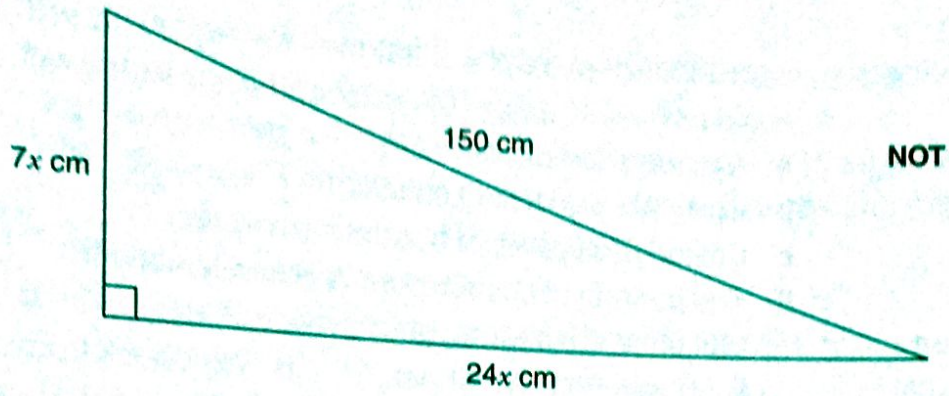
In triangle PQR, angle QPR is acute, $PQ = 10$ cm and $PR = 14$ cm.

a The area of triangle PQR is 48 cm^2.
Calculate angle QPR and show that it rounds to 43.3°, correct to 1 decimal place. You must show all your working.

b Calculate the length of the side QR.

(0580 paper 04 Q3 June 2009)

17



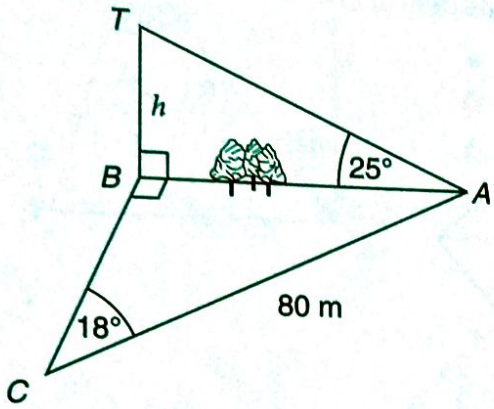
NOT TO SCALE

The right-angled triangle shown in the diagram has sides of length $7x$ cm, $24x$ cm and 150 cm.

a Show that $x^2 = 36$.

b Calculate the perimeter of the triangle. (0580 paper 02 Q10 November 2006)

18



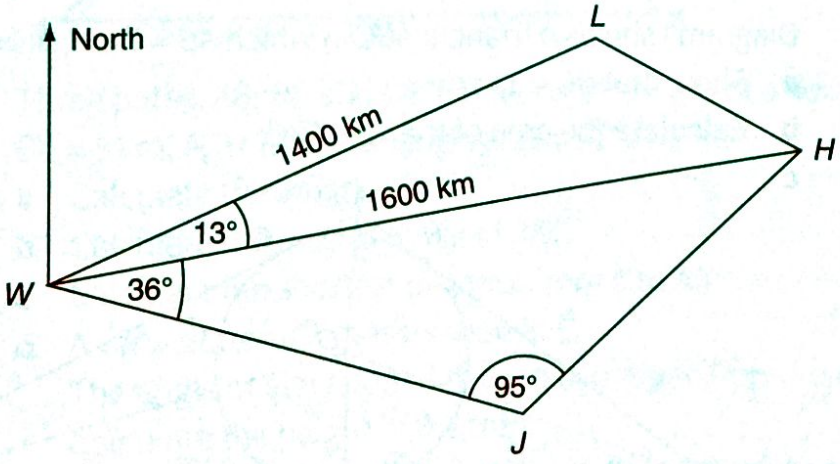
NOT TO SCALE

Mahmoud is working out the height, h metres, of a tower BT which stands on level ground.
He measures the angle TAB as 25°.
He cannot measure the distance AB and so he walks 80 m from A to BT, where angle ACB = 18° and angle ABC = 90°.
Calculate

a the distance AB,

b the height of the tower, BT. (0580 paper 21 Q15 June 2009)

19



NOT TO SCALE

The diagram shows the positions of four cities in Africa, Windhoek (W), Johannesburg (J), Harari (H) and Lusaka (L).

$WL = 1400$ km and $WH = 1600$ km.
Angle $LWH = 13°$, angle $HWJ = 36°$ and angle $WJH = 95°$.

a Calculate the distance LH.

b Calculate the distance WJ.

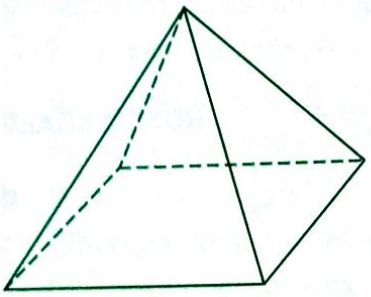
c Calculate the area of quadrilateral $WJHL$.

d The bearing of Lusaka from Windhoek is 060°.
Calculate the bearing of

i Harari from Windhoek, **ii** Windhoek from Johannesburg.

e On a map the distance between Windhoek and Harari is 8 cm.
Calculate the scale of the map in the form $1 : n$. (0580 paper 04 Q2 November 2006)

20 The base of a pyramid is a square with diagonals of length 6 cm.
The sloping faces are isosceles triangles with equal sides of length 7 cm.
The height of the pyramid is $\sqrt{l}$ cm.
Calculate l.



(4024 paper 01 Q18 June 2004)

21

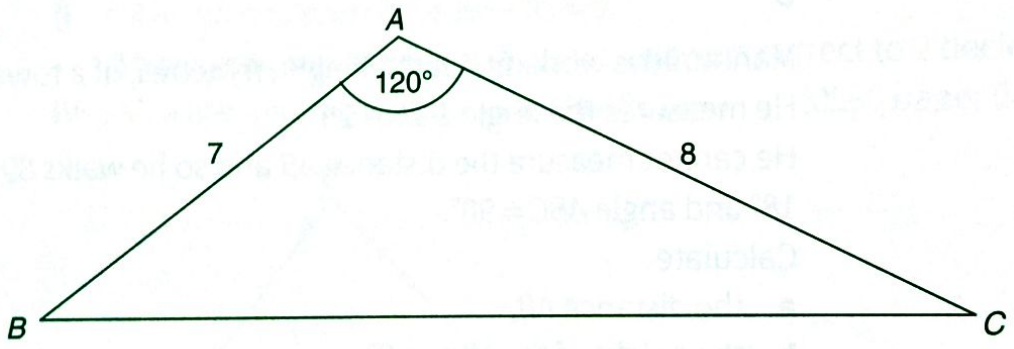


Diagram I

Diagram I shows a triangle ABC in which $AB = 7$ cm, $AC = 8$ cm and $B\hat{A}C = 120°$.

a Show that $BC = 13$ cm.

b Calculate the area of triangle ABC.

c

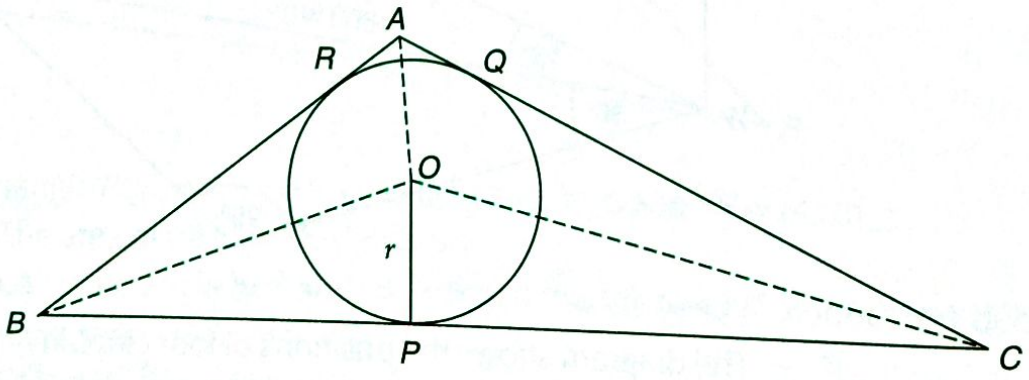


Diagram II

The sides of the triangle ABC, shown in Diagram I, are tangents to a circle with centre O and radius r centimetres.

The circle touches the sides BC, CA and AB at P, Q and R respectively, as shown in Diagram II.

i Find an expression, in terms of r, for the area of triangle OBC.

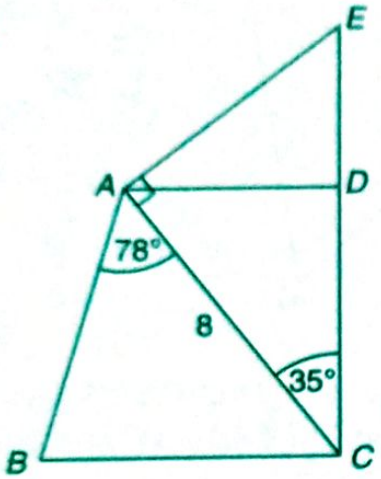
ii By similarly considering the areas of triangles OAB and OAC, find an expression, in terms of r, for the area of triangle ABC.

iii Hence find the value of r.

d Calculate the percentage of the area of triangle ABC that is **not** occupied by the circle.

(4024 paper 02 Q9 June 2004)

22



The diagram represents some beams which support part of a roof.

AD and BC are horizontal and CDE is vertical.

$AC = 8$ metres, $B\hat{A}C = 78°$, $A\hat{C}D = 35°$ and $C\hat{A}E = 90°$.

Calculate the length of the beam

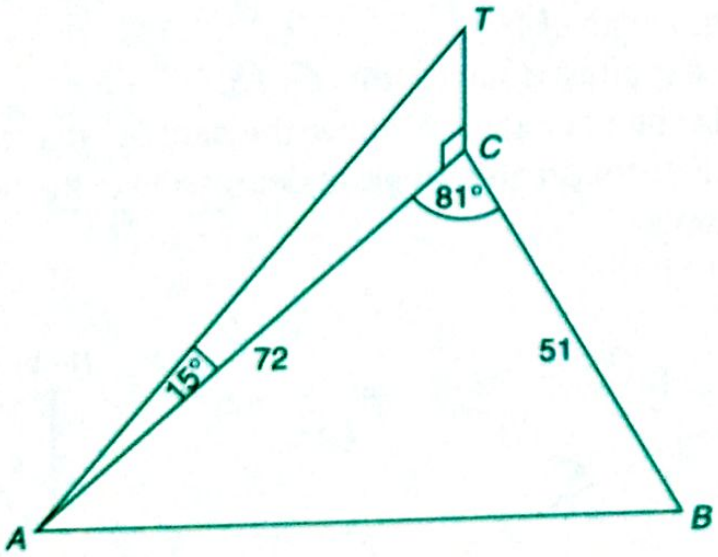
a AD,

b CE,

c AB.

(4024 paper 02 Q1 November 2004)

23



Three paths, AB, BC and CA, run along the edges of a horizontal triangular field ABC.

$BC = 51$ m, $AC = 72$ m and angle $ACB = 81°$.

a Calculate the length of AB.

b Calculate the area of the field ABC.

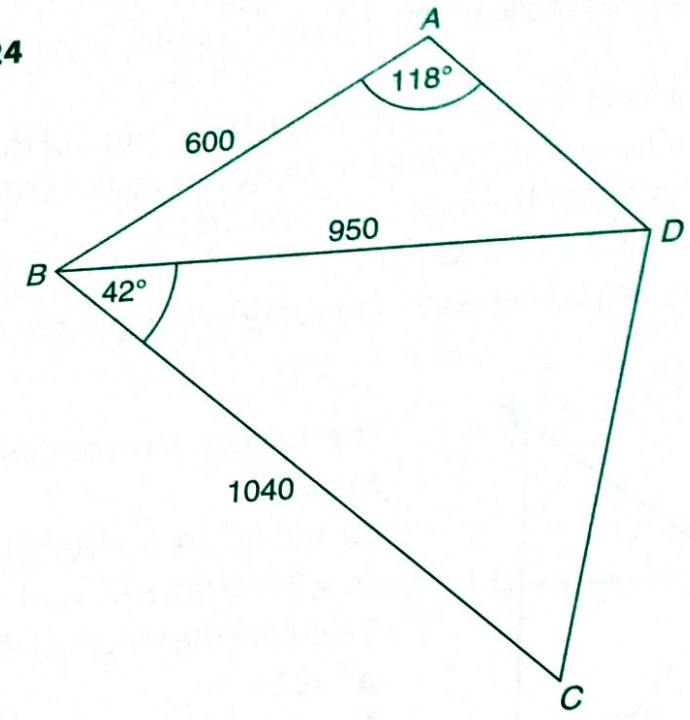
c Calculate the shortest distance from C to AB.

d A vertical tree, CT, has its base at C.
The angle of elevation of the top of the tree from A is 15°.
Calculate the height of the tree.

e John measured the largest angle of elevation of the top of the tree as seen from the path AB.
Calculate this angle of elevation.

(4024 paper 02 Q7 November 2004)

24



In the diagram, the quadrilateral $ABCD$ represents a level park with a path BD.
$AB = 600$ m, $BC = 1040$ m, $BD = 950$ m, $C\hat{B}D = 42°$ and $B\hat{A}D = 118°$.

a Calculate

i angle ABD,

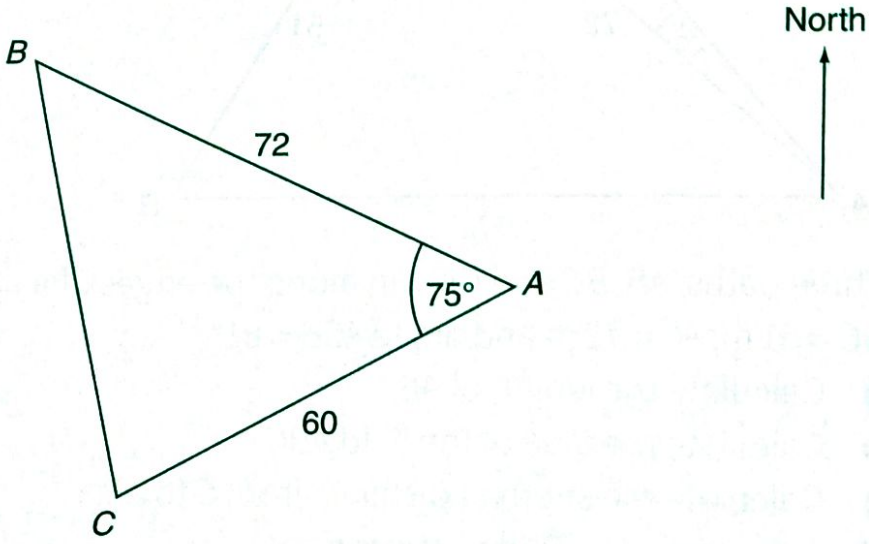
ii the length of CD,

iii the shortest distance from C to BD.

b A helicopter flew directly above the path BD at a constant height of 500 m.
Calculate the greatest angle of depression of the point C as seen by a passenger on the helicopter.

(4024 paper 02 Q9 June 2005)

25



Three points, A, B and C, lie on a horizontal field.
Angle $BAC = 75°$ and the bearing of C from A is 217°.
$AB = 72$ m and $AC = 60$ m.

a Calculate

i the bearing of B from A,

ii BC,

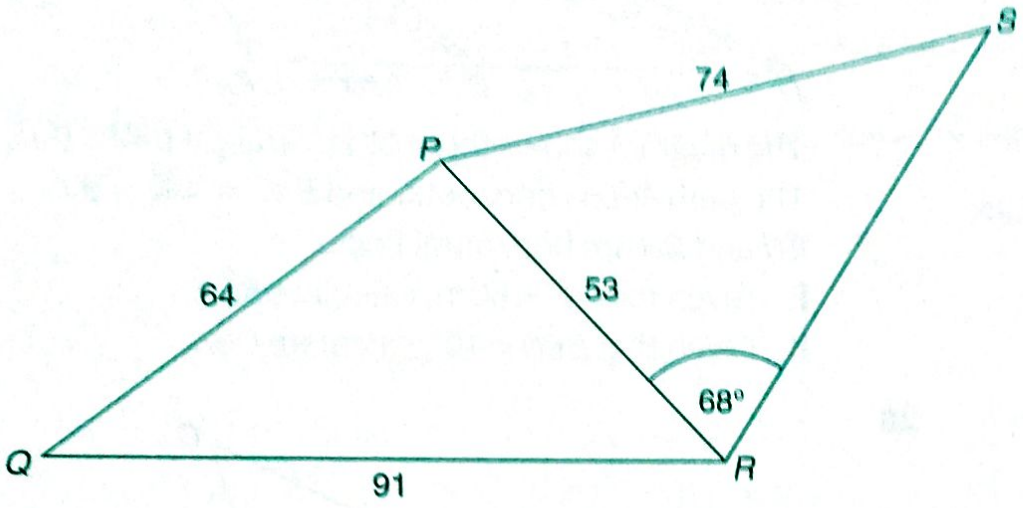
iii angle ABC,

iv the bearing of C from B.

b A girl standing at B is flying a kite.
The kite, K, is vertically above A.
The string, BK, attached to the kite is at 24° to the horizontal.
Calculate the angle of elevation of the kite when viewed from C.

(4024 paper 02 Q8 November 2005)

16



The diagram shows a footpath PR across a park $PQRS$.
$PQ = 64$ m, $PR = 53$ m, $PS = 74$ m and $QR = 91$ m.
Angle $PRS = 68°$.
Calculate

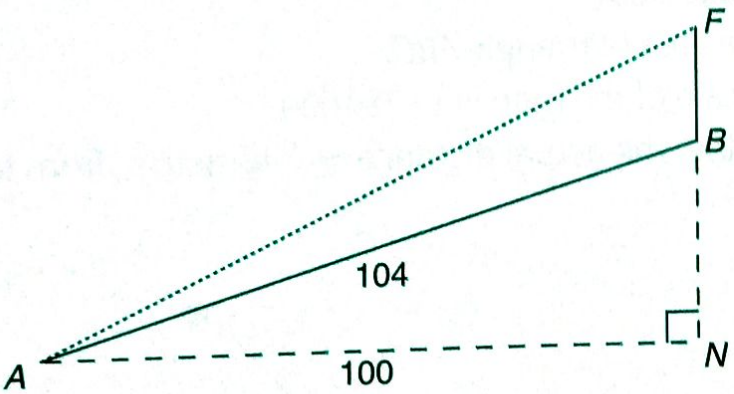
a $Q\hat{P}R$,

b $R\hat{P}S$,

c the area of triangle PRS.

(4024 paper 02 Q3 November 2006)

17 A vertical flagpole, BF, stands at the top of a hill.
AB is the steepest path up the hill.
N lies vertically below B and $A\hat{N}B = 90°$.
$AN = 100$ m and $AB = 104$ m.



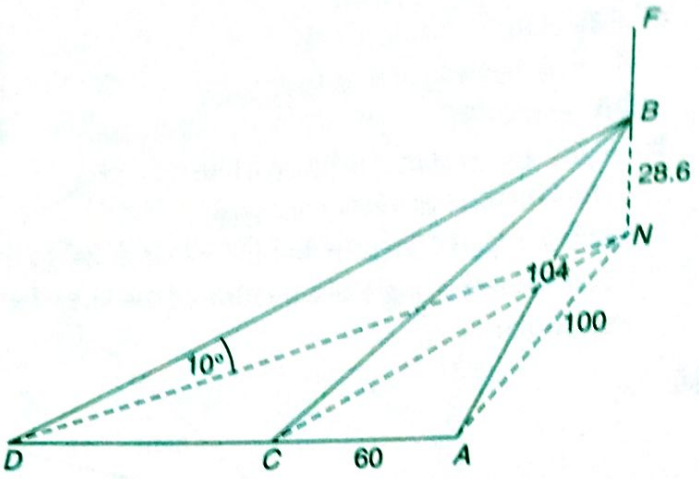
a Show that $BN = 28.6$ m.

b It is given that $F\hat{A}N = 25°$.

i Write down the size of the angle of depression of A from F.

ii Calculate the height, BF, of the flagpole.

c

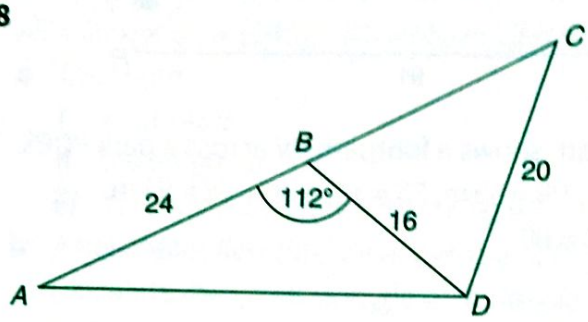


The diagram shows three other straight paths (*CB*, *DB* and *ACD*) on the hill.
The path *ACD* is horizontal and $B\hat{A}C = N\hat{A}C = 90°$.
CN and *DN* are horizontal lines.

i Given that $AC = 60$ m, calculate $B\hat{C}N$.

ii Given that $B\hat{D}N = 10°$, calculate $D\hat{B}A$. (4024 paper 02 Q9 November 2006)

28



The points *A*, *B*, *C* and *D* represent four towns on a map.
ABC is a straight line.
$AB = 24$ cm, $BD = 16$ cm and $CD = 20$ cm.
Angle $ABD = 112°$.

a Calculate

i *AD*,

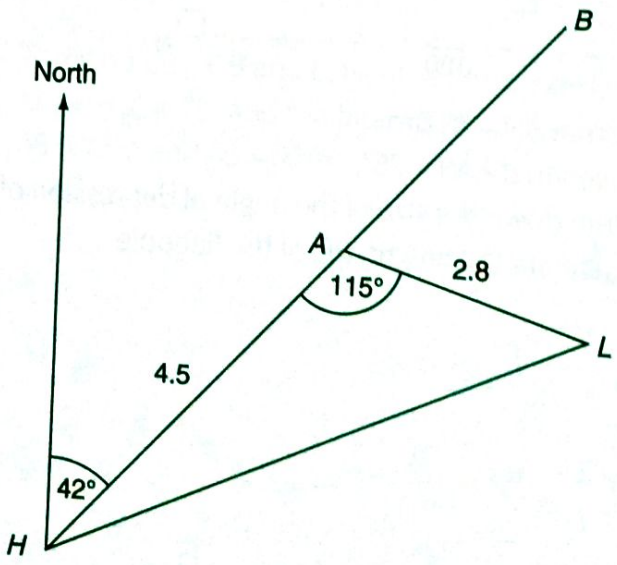
ii angle *BCD*,

iii the area of triangle *ABD*.

b The scale of the map is 1 : 250 000.
Calculate the actual distance, in kilometres, from town *A* to town *B*.

(4024 paper 02 Q5 November 2007)

29



The diagram shows the positions of a harbour, H, a lighthouse, L, and two buoys A and B.
HAB is a straight line.
The bearing of A from H is 042°.
$HA = 4.5$ km, $AL = 2.8$ km and $H\hat{A}L = 115°$.

a Find the bearing of

i H from A,

ii L from A.

b Calculate

i HL,

ii the area of triangle HAL.

c A boat sailed from the harbour along the line HAB.

i Calculate the shortest distance between the boat and the lighthouse.

ii The boat sailed at a constant speed of 3 m/s.

Given that the boat reached A at 0715, find at what time it left the harbour.

(4024 paper 02 Q9 June 2008)

6 **a** 43.1 cm

b $\frac{1}{2} \times 61 \times 30 \times \sin 41 = 600$

c The two triangles have the same height, and bases in the ratio

$45 : 30 = 3 : 2$.

NOTE:

Drop a perpendicular from B to the line CA. This is the height of both triangles.

d 900 m^2 **e** 41.7 to 41.9 m **f** 21.0 to 21.1°

8 **a** $\frac{1}{2} \times 40.3 \times 26.8 \times \sin 92 = 540 cm^2$

b $\frac{AB}{\sin 92} = \frac{40.3}{\sin 55}$

$AB = \frac{40.3 \times \sin 92}{\sin 55} = 49.2 cm$

c 55° (angles in the same segment)

d 33°

e similar

f $\frac{XD}{40.3} = \frac{20.1}{26.8}$

$XD = 30.2$ cm

9 **a** 0.176 **b** 1.76×10^{-1}

10 1

11 **a** 0 **b** −1.5

c It is below the height at midday

12 **i** $(2y - 1)^2 = y^2 + (y + 2)^2$

$4y^2 - 4y + 1 = y^2 + y^2 - 4y + 4$

$2y^2 - 8y - 3 = 0$

ii −0.35 or 4.35 **iii** 13.8

13 **a** 59.2 m **b** 2360 m² **c** 129 m² **d** 31.9°

14 7.94

15 60, 120

16 **a** **i** $\frac{1}{2}x(x+4)=48$

ii −12 or 8 **iii** 12 cm

b $\frac{4}{5}$ or 0.8

c **i** $(x+4)^2+x^2=9^2$

$x^2+8x+16+x^2=81$

ii $x=\frac{-8\pm\sqrt{64+4\times2\times65}}{4}$

$x=-8.04$ or 4.04

iii 21.1 cm

17 **a** $\frac{1}{2}\times10\times14\times\sin P=48$

$\sin P=\frac{48}{70}$, $P=43.29=43.3°$ to 1 dp

b 9.60 cm

18 **a** $(7x)^2+(24x)^2=150^2$

$625x^2=22500$, $x^2=36$

b 336 cm

19 **a** 24.7 m **b** 11.5 m

20 **a** 393 to 393.5 km **b** 1210 km

c 820900 to 822000 km²

d **i** 073° **ii** 289°

e 1:20 000 000

21 $l=40$

22 **a** Hint: use the Cosine Rule

b 24.2 cm²

c **i** **ii** $14r$ **iii** $r=1.73$

d 61.1%

23 **a** 4.59 m **b** 9.77 m **c** 8.96 m

24 **a** 81.5 m **b** 1810 m² **c** 44.5 m

d 19.3 m **e** 23.4°

25 **a** **i** 28.1° **ii** 718 m **iii** 696 m

b 35.7°

26 **a** **i** 292° **ii** 80.9 m **iii** 45.70 to 45.80°

iv 157.70 to 158°

b 28.1°

27 **a** 101.7° **b** 70.4° **c** 1850 m²

28 **a** Hint: use Pythagoras

b **i** 25° **ii** 18.00 to 18.10 m

c **i** 13.70 to 13.80 **ii** 50.75 to 50.85

29 **a** **i** 33.5 cm **ii** 47.9° **iii** 178 cm²

b 60 km

30 **a** **i** 222° **ii** 107°

b **i** 6.22 km **ii** 5.71 km²

c **i** 2.54 km **ii** 0650